PART I.

Containing

The Properties of Bodies.

Their Laws of Motion.

AND

The Mechanical Powers.

The Second Edition.

[Price of this Part 1s. 6d.]

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A

Compendious System

OF

Natural Philosophy.

With NOTES

Containing the

Mathematical Demonstrations,

AND

Some Occasional Remarks.

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CAMBRIDGE,

Printed for the Author at the University-Press; and sold by W. Innys and R. Manby, at the West End of St. *Paul*'s; S. Harding in St. *Martin*'s Lane, London: R. Clements in Oxford: C. Crownfield and W. Thurlbourn in Cambridge. M,DCC,XXXV.

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A COMPENDIOUS SYSTEM OF Natural Philosophy.

PART I.

The INTRODUCTION.

SO wild and extravagant have been the Notions of a great Part of Philoſophers, both Ancient and Modern, that it is hard to determine, whether they have been more diſtant in their Sentiments from Truth, or from one another; or have not exceeded the Fancies of the moſt fabulous Writers, even Poets and Mythologiſts. This was owing to a precipitate Proceeding in their ſearching into Nature, their neglecting the uſe of Geometry and Experiment, the moſt neceſſary Helps to the finding out Cauſes, and proportioning them to their Effects.

THE Manner of Philoſophizing among the Ancients was to give Bodies certain *arbitrary*

Properties, ſuch as beſt ſerv'd their Purpoſe in accounting for the Phænomena * of Nature; from whence proceeded ſo many various Sects of Philoſophers; every one aſcribing a different Cauſe to the ſame Appearance, as his particular Genius and Imagination led him.

THE chief Agreement obſervable among moſt of them, conſiſts in this, *viz.* that they conceived all Bodies, as Compoſitions of Air, Earth, Fire, and Water, or ſome one or more of them, from whence they acquired the Name of Principles or Elements, which they ſtill retain.

EPICURUS advanc'd a little farther, and aſſerted, that though Bodies conſiſted of ſome one or more of theſe, yet that they were not ſtrictly Elements, but that theſe themſelves conſiſted of Atoms; by an accidental Concourſe of which, (as they were moving through infinite Space in Lines nearly parallel) all Things received their Form and Manner of Exiſtence †.

DES CARTES has contrived an Hypotheſis very different from the reſt; he ſets out with a

* By a Phænomenon of Nature is meant any Motion or Situation of Bodies among one another, which offers it ſelf to the Notice of our Senſes, and is not the immediate Reſult of the Action of an intelligent Being.

† For the Opinions of the Ancient Philoſophers conſult *Diogenes Laertius* and *Stanley*'s Lives.

Suppo-

Suppoſition that the Univerſe at firſt was entirely full of Matter, that, from this Matter when firſt put in Motion, there would neceſſarily be rubbed off (by the grinding of the ſeveral Parts one againſt another) ſome Particles ſufficiently fine to paſs through the hardeſt and moſt ſolid Bodies without meeting with any Reſiſtance: of theſe conſiſts his *Materia ſubtilis*, or *Materia primi Elementi*. He imagined that from hence alſo would reſult other Particles of a globular Form, to which he gave the name of *Materia ſecundi Elementi*. Thoſe which did not ſo far loſe their firſt figure, as to come under the Denomination of *Materia primi* or *ſecundi Elementi*, he call'd *Materia tertii Elementi*; and maintain'd that all the Variety, which appears in natural Bodies, was owing to different Combinations of thoſe Elements.

He likewiſe ſuppoſes that God created a certain Quantity of Motion, and aſſigned it to this Maſs of Matter, and that That Motion (being once created) could no more be annihilated without an omnipotent Hand, than Body it ſelf; in Conſequence of which, he was obliged to teach, that the Quantity of Motion is always the ſame: So that if all the Men and Animals in the World were moving, yet ſtill there would be no more Motion, than when they were at Reſt; the Motion which they had not when at Reſt, being

transferr'd to the Æther. So unaccountable are the Notions of this great Philosopher, that it is surprizing his Doctrine should have met with such universal Reception, and have got so strong a Party of Philosophers on its Side, that notwithstanding it was more absurd, than the Schoolmens *Substantial Forms*, they must all be exploded to make Way for his ingenious Hypothesis.

Des Cartes has been said by a late Writer *, to have joined to his great Genius an exquisite Skill in Mathematics, and by mixing Geometry and Physics together, to have given the World Hopes of great Improvements in the latter. But this Writer ought to have considered that what he look'd upon in Des Cartes's Book of Principles, as Demonstrations, are only Illustrations, there not being a Demonstration from Geometry in all his Philosophical Works †.

The present Method of Philosophizing establish'd by Sir Isaac Newton, is to find out the Laws of Nature by Experiments and Observations. To this, with a proper Applica-

* Mr. *Wotton* in his Reflections on Ancient and Modern Learning.

† See this Subject discuss'd more at large in *Keil*'s Introduction to his Examination of Dr. *Burnet*'s Theory.

tion

tion of Geometry, is owing the great Advantage the preſent Syſtem of Philoſophy has over all the preceeding ones, and the vaſt Improvement it has received within the laſt Age. It is indeed in vain to imagine, that a Syſtem of Natural Philoſophy can be framed by any other Method: for without Obſervations it is impoſſible we ſhould diſcover the Phænomena of Nature, without Experiments we muſt be ignorant of the mutual Actions of Bodies, and without Geometry we can never be certain whether the Cauſes aſſign'd be adequate to the Effects we would explain, as the various Syſtems of Philoſophy built on other Foundations, evidently ſhew.

THIS Way of ſearching into Nature was firſt propoſed by my Lord BACON *, proſecuted by the *Royal Society*, the *Royal Academy* at *Paris*, the Honourable Mr. BOYLE, Sir ISAAC NEWTON, *&c.*

WHAT wonderful Advancement in the Knowledge of Nature may be made by this Method of Enquiry, when conducted by a Genius equal to the Work, will be beſt underſtood by conſidering the Diſcoveries of that excellent Philoſopher laſt mentioned. To Him it is principally owing, that we have now a

* See his *Novum Organum.*

rational

rational Syſtem of Natural Philoſophy; 'tis He who, by purſuing the ſure and unerring Method of Reaſoning from Experiment and Obſervation, joined with the moſt profound Skill in Geometry, has carried his Enquiries to the moſt minute and inviſible Parts of Matter, as well as to the moſt remote Bodies in the Univerſe, and has eſtabliſh'd a Syſtem, not ſubject to the uncertainty of a mere Hypotheſis, but which ſtands upon the ſecure Baſis of Geometry it ſelf.

CHAP.

CHAP. I.

The Properties of Body.

IT being the Deſign of *Phyſics* or Natural Philoſophy to account for the Phænomena of the Material World, it is neceſſary to begin with laying down the known Properties of Body.

THESE are 1. Solidity. 2. Extenſion. 3. Diviſibility. 4. A Capacity of being moved from Place to Place. 5. A Paſſiveneſs or Inactivity. Which are the eſſential Properties of Body; as appears from what follows.

1. SOLIDITY, called alſo Impenetrability, is that Power which Body has of excluding all others out of its Place.

THAT Body, as ſuch, muſt be endued with this Property follows from its Nature, for otherwiſe two Bodies might exiſt in the ſame Place, which is abſurd. The ſofteſt are equally Solid with the hardeſt, for we find by Experiment, that the Sides of a Bladder filled with Air or Water, can by no means be made to come cloſe together *.

* At *Florence* a hollow Globe of Gold was fill'd with Water, and then exactly clos'd; the Globe thus clos'd was put into a Preſs driven by the Force of Screws; the Water finding no Room for a nearer Approach of its Particles toward each other, made its Way through the Pores of that cloſe

Metal

2. THAT Body is extended, is ſelf evident, it being impoſſible to conceive any Body, which has not Length, Breadth and Thickneſs, that is, Extenſion.

3. IT is no leſs evident, that Body is diviſible, for ſince no two Particles of Matter can exiſt in the ſame Place, it follows that they are really diſtinct from each other, which is all that is meant by being diviſible.

IN this Senſe the leaſt conceivable Particle muſt ſtill be diviſible, ſince it will conſiſt of Parts, which will be really diſtinct *. To illuſtrate this by a familiar Inſtance: Let the leaſt imaginable Piece of Matter be conceived lying on a ſmooth plane Surface, 'tis evident the Surface will not touch it every where, thoſe Parts therefore, which it does not touch, may be ſuppoſed ſeparable from the other, and ſo on as far as we pleaſe; and this is all that is

Metal ſtanding in Drops like Dew on the outſide, before the Globe would yield to the violent Preſſure of the Engine. V. *Acad. del Ciment.*

* This Propoſition is demonſtrated Geometrically thus, ſuppoſe the Line *AD* (*Fig.* 1.) perpendicular to *BF* and another as *GH* at a ſmall Diſtance from it alſo perpendicular to the ſame Line; with the Centers *CCC* &c. deſcribe Circles cutting the Line *GH* in the Points *e*, *e*, *e*, &c. Now the greater the Radius *AC* is, the leſs is the Part *eH*. But the Radius may be augmented in infinitum, and therefore the Part *eH* may be diminiſhed in the ſame Manner; and yet it can never be reduc'd to nothing, becauſe the Circle can never coincide with the right Line *AF*; conſequently the Parts of any Magnitude repreſented by *GH* may be diminiſhed in infinitum. *Q. E. D.* V. *Keil*'s Introd. ad Phyſ. Præl 3, 4, 5. *Graveſande*'s Elem. Math. Phyſ. L. 1. c. 4. Schol.

is meant, when we ſay Matter is infinitely diviſible.

How far Matter may actually be divided, may in ſome manner be conceiv'd from hence * that a Piece of Wire gilt with ſo ſmall a Quantity as eight Grains of Gold, may be drawn out to the Length of thirteen Thouſand Feet, the whole Surface of it ſtill remaining cover'd with Gold †.

A Quantity of Vitriol being diſſolved and mix'd with nine Thouſand Times as much Water, will tinge the whole, conſequently the Vitriol will be divided into as many Parts as there are viſible portions of Matter in that Quantity of Water ‡.

THERE are Perfumes, which, without a ſenſible Diminution of their Quantity, ſhall fill a very large Space with their odoriferous Particles, which muſt therefore be of an inconceivable ſmallneſs, ſince there will be a ſuffici-

* We have a ſurprizing Inſtance of the minuteneſs of ſome Parts of Matter from the Nature of Light and Viſion. Let a Candle be lighted and placed in an open Plane, it will then be viſible two Miles round, conſequently was it placed two Miles above the Surface of the Earth, it would fill with luminious Particles a Sphere, whoſe diameter was four Miles, and that before it had loſt any ſenſible part of its Weight. The Force of this Argument will appear better when the Reader is acquainted with the Cauſe of Viſion.

† *Keil's* Introd. ad Phyſ. Præl. 5. Religious Philoſ. Contempl, 25.

‡ Mem. de l' Acad. 1706.

ent Number in every Part of that Space, sensibly to affect the Organ of Smelling.

4. THAT all Matter is moveable, follows from its being Finite: and to suppose it positively Infinite is absurd, because it consists of Parts *.

5. BY the Passiveness or Inactivity of Matter, (commonly call'd its *Vis Inertiæ*) is meant the Propensity it has to continue its State of Motion or Rest, till some external Force acts upon it. This will be farther explain'd under the first Law of Nature.

CHAP. II.

Of Vacuum.

I. PLACE void of Matter is call'd empty Space or *Vacuum.*

II. IT has been the Opinion of some Philosophers, particularly the *Cartesians*, that Nature admits not a *Vacuum*, but that the Universe is entirely full of Matter: in consequence of which Opinion they were oblig'd to assert, that if every Thing contain'd in a Vessel could be taken out or annihilated, the Sides of that Vessel, however strong, would come together; but this is contrary to Experience,

* See Mr. *Law's* Translation of ABp. *King de Origine Mali.* Note 3.

rience, for the Air may be drawn out of a Veſſel by means of the Air Pump, which will nevertheleſs remain whole, if its Sides are ſtrong enough to ſupport the Weight of the incumbent Atmoſphere.

III. SHOULD it be objected here, that it is impoſſible to extract all the Air out of a Veſſel, and that there will not be a Vacuum on that Account; the Anſwer is, that ſince a very great Part of the Air that was in the Veſſel, may be drawn out, as appears by the more quick deſcent of light Bodies in a Receiver * when exhauſted of its Air, there muſt be ſome Vacuities between the Parts of the remaining Air: which is ſufficient to conſtitute a *Vacuum*. Indeed to this it may be objected by a *Carteſian*, that thoſe Vacuities are fill'd with *Materia ſubtilis*, that paſſes freely through the Sides of the Veſſel, and gives no Reſiſtance to the falling Bodies: but as the Exiſtence of this *Materia ſubtilis* can never be prov'd, we are therefore not oblig'd to allow the Objection; eſpecially ſince Sir ISAAC NEWTON has found, that all Matter affords a Reſiſtance nearly in Proportion to its Denſity †.

THERE are many other Arguments to prove this, particularly the Motions of the Comets

* By this Term is meant any Veſſel, out of which we extract the Air by the Air Pump.

† *Newt.* Principia Lib. 2 Prop. 31. & 40. & Opt. Edit. 2. Book 3. Quer. 18, 19, 20, 21. *Deſagul.* Lect. 1. Ann. 2.

through the Heavenly Regions without any ſenſible Reſiſtance *; the different Weight of Bodies of the ſame Bulk *&c.* but thoſe, being not yet explain'd, are not ſo proper to be inſiſted on in this Place.

CHAP. III.

Of Attraction and Repulsion.

I. BESIDES the forementioned Properties of Matter, it has alſo certain Powers or active Principles, known by the Names of *Attraction* and *Repulſion*, probably not eſſential or neceſſary to its Exiſtence, but impreſſed upon it by the Author of its Being, for the better Performance of the Offices for which it was deſign'd.

II. ATTRACTION is of two Kinds. 1. Coheſion, or that by which minute Bodies, (or the ſeveral Particles of the ſame Body) when placed aſunder at very ſmall Diſtances, mutually approach each other; and then adhere or ſtick together, as if they were but one. 2. Gravitation, or that by which diſtant Bodies act upon each other.

III. THE Attraction of Coheſion is prov'd from abundance of Experiments, of which ſome of the moſt obvious are as follows.

* *Deſagul.* Lect. 1. Annot. 8.

1. LET

1. LET a ſmall glaſs Tube (commonly call'd a Capillary Tube) open at both Ends, be dipt into a Veſſel of Water, the Water will immediately riſe up in the Tube to a certain Height above the Level. This riſe of the Water is manifeſtly owing to the Attraction of thoſe Particles of the Glaſs, which lie in the inner Surface of the Tube immediately above the Water: accordingly the Quantity of Water raiſed is always proportionable to the largeneſs of that Surface*.

2. LET two Spheres of Quickſilver be placed near each other, and they will immediately run together, and form one Globule.

IV. THE Laws of this Attraction are 1*ſt*. That it acts only upon Contact, or at very ſmall Diſtances; for the Spheres mentioned in the laſt Experiment, will not approach each other, till they are plac'd very near. 2. It

* The Heights the Water riſes to in different Tubes, are obſerv'd to be reciprocally as the Diameters of the Tubes, from whence it follows that the Quantities raiſed are as the Surfaces which raiſe them.

Dem. Let there be two Tubes, the Diameter of the firſt double to that of the ſecond, the Water will riſe half as high in the firſt as in the ſecond, now was it to riſe equally high in both, the Quantity in the firſt would be four times as great as in the ſecond, (Cylinders of equal Heights being as the Squares of their Diameters; 11. *El.* 14.) therefore ſince it is found to riſe but half as high, the Quantity is but twice as much, and therefore as the Diameter; but the Surfaces of Cylinders are as their Diameters, therefore the Quantities of Water raiſed are alſo as the Surfaces. *Q. E. D.*

See the Diſſertation on this Subject. Part II.

acts

acts according to the Breadth of the Surfaces of the Attracting Bodies, and not according to their Quantities of Matter. For, let there be two polish'd Glaſs Plates laid one upon another, in ſuch a Manner, as to touch at one End, and there make a very ſmall Angle: if two unequal Drops of Oil be put between theſe Plates, at equal Diſtances from the Line of Contact, ſo that the leaſt may touch both Glaſſes, they will then both move towards the Ends that touch, becauſe the Attraction of the Surfaces inclines that Way; but the largeſt, touching the Glaſſes in moſt Points, will move the faſteſt. 3. 'Tis obſerv'd to decreaſe much more than as the Squares of the Diſtances of the Attracting Bodies from each other increaſe: that is, whatever the Force of Attraction is at a given Diſtance, at twice that Diſtance it ſhall be more than four Times leſs than before *.

V. FROM hence it is eaſy to account for the different Degrees of Hardneſs in Bodies; thoſe whoſe conſtituent Particles are flat or ſquare, and ſo ſituated as to touch in many Points, will be hard; thoſe Particles which are more round, and touch in fewer Points, will conſtitute a ſofter Body; thoſe which are ſpherical, or nearly of that Figure, will form a Fluid †.

* V. *Keilii* Opera Ed. *4to.* p. 626.

† See *Rohault* in the Notes, pag. 105, 108. See Part II. Chap. I. §. 2. in the Notes. *Newtoni* Optic. p. 335.

VI. Attraction of Gravitation is that, by which diſtant Bodies act upon each other. Of this we have daily Inſtances in the falling of heavy Bodies toward the Earth.

VII. The Laws of this Attraction are 1. That it decreaſes, as the Squares of the Diſtances between the Centers of the attracting Bodies increaſe. Thus, a Body which at the Surface of the Earth (*i. e.* about the Diſtance of four Thouſand Miles from its Center,) weighs ten Pounds, if it was plac'd four Thouſand Miles above the Surface of the Earth *i. e.* twice as far diſtant from the Center as before, would weigh four Times leſs; if thrice as far, nine Times leſs *&c.* The Truth of this Propoſition is not to be had from Experiments, (the utmoſt Diſtance we can convey Bodies to, from the Surface of the Earth, bearing no Proportion to their Diſtance from its Center,) but is ſufficiently clear from the Motions obſerv'd by the Heavenly Bodies. 2. Bodies attract one another with Forces proportionable to the Quantities of Matter they contain; for all Bodies are obſerv'd to fall equally faſt in the exhauſted Receiver, where they meet with no Reſiſtance. From whence it follows, that the Action of the Earth upon Bodies is exactly in Proportion to the Quantities of Matter they contain; for was it to act as ſtrongly upon a leſs Body as upon a larger, the leaſt Body, being moſt eaſily put into Motion,

would

would move the fasteſt. Accordingly, it is obſervable, that the Weight of a Body is the ſame, whether it be whole, or ground to Powder *.

VIII. From hence it follows, that, was a Body to deſcend from the Surface toward the Center of the Earth, it would continually become lighter and lighter, the Parts above attracting it, as well as thoſe below; in which Caſe it is demonſtrated by Mathematicians, that the Gravity would decreaſe with the Diſtance of the Body from the Center †.

* *Graveſande* Lib. 4. Chap. 11. *Cotes's* Preface to *Newton's* Princip.

† *Dem.* Let there be a Body as *P*, (*Fig.* 2.) placed any where within a concave Sphere, as *AB*, which let us ſuppoſe divided into an infinite Number of thin concentric Surfaces; I ſay, the Body *P* will be attracted equally each way by any one of theſe, *v. g.* the interior *HIKLM* Let there be Lines as *IL*, *HK*, &c. drawn through any Point of the Body *P*, in ſuch a manner as to form the Surface of two ſimilar Figures, ſuppoſe Cones, the Diameters of whoſe Baſes may be *IH*, *KL*, which let be infinitely ſmall. Theſe Baſes (being as the Squares of the Lines *IH*, *KL*) (2. *Elem.* 12) will be directly, as the Squares of their Diſtances from *P* (for the Triangles *IPH*, *KPL* being infinitely ſmall, are ſimilar.) But thoſe Baſes include all the Particles of Matter in the interior Surface, that are oppoſite to each other; the oppoſite Attractions are therefore in the ſame ratio with thoſe Baſes, that is as the Squares of the diſtances *PK*, *PI* But the attraction is inverſely, as the Squares of the Diſtances of the attracting Bodies, §. 7. *i. e.* inverſely as the Squares of the ſame Diſtances *PK*, *PI*; theſe two ratios therefore deſtroying each other, it is evident, that if the Concavity of the Sphere was filled with Matter, that alone, which lies nearer the Center than the Body can effect it, the reſpective Actions of all the Parts, that are more diſtant, being equal, and

Scholium. It may be proper to obſerve here, that when Philoſophers ſpeak of Bodies gravitating to, or attracting each other, that Body is ſaid to gravitate to another, which moves towards it, while the other actually is or appears to be, at reſt, and this other is ſaid to attract the former; though indeed the force being mutual and equal on both Sides (as will be explain'd under the third Law of Nature) the ſame Term might be apply'd to either the gravitating or attracting Body.

It is farther to be obſerv'd, that when we uſe the Terms, Attraction or Gravitation, we do not thereby determine the Phyſical Cauſe of it, as if it proceeded from ſome ſuppoſed *occult* Quality in Bodies; but only uſe thoſe Terms to ſignify an Effect, the Cauſe of which lies out of the reach of our Philoſophy. Thus, we may ſay, that the Earth Attracts heavy Bodies; or that ſuch Bodies tend or gravitate to the Earth: although at the ſame time we

and in contrary Directions, ſince the ſame is demonſtrable of any of the remaining concentric Surfaces. Let us ſee then what effect that, which lies nearer the Center than the Body, will have upon it, which may be conſidered as a Sphere, on whoſe Surface the Body is plac'd. The Diſtances of each Particle of Matter from the Body, (taken collectively) will be as the Diameter of the Sphere, or as the Radius, *i. e.* as the Diſtance of the Body from the Center: their action therefore upon the Body will be inverſely as the Square of that Diſtance: but the Quantity of Matter will be as the Cube of that Diſtance; (18. *Elem.* 12) the Attraction therefore will be alſo in that Proportion. Now, theſe two Ratios being compounded, the Attraction will be only as the Diſtance of the Body from the Center. *Q. E. D.*

are wholly ignorant, whether this is effected by ſome Power, actually exiſting in the Earth, or in the Bodies, or external to both; ſince it is impoſſible any Error in our Reaſonings can follow from hence: it being evident, that all the Conſequences of ſuch tendency muſt be the ſame, let the Cauſe be where, or what it will.

X. REPULSION is that Property in Bodies, whereby, if they are placed juſt beyond the Sphere of each others Attraction of Coheſion, they mutually fly from each other,

THUS, if an oily Subſtance, lighter than Water, be placed on the Surface thereof, or if a Piece of Iron be laid on Mercury, the Surface of the Fluid will be depreſs'd about the Body laid on it: This Depreſſion is manifeſtly occaſion'd by a repelling Power in the Bodies, which hinders the Approach of the Fluid towards them.

BUT it is poſſible in ſome Caſes to preſs or force the repelling Bodies into the Sphere of one anothers Attraction; and then they will mutually tend towards each other; as when we mix Oyl and Water till they incorporate *.

XI. BESIDES the general Powers forementioned, there are ſome Bodies, that are endued with another, call'd *Electricity*. Thus, Amber,

* We have an undeniable Proof of this Repulſive Force in Sir *Iſaac Newton's* Opticks. B. 3. and Query 31.

Jet,

Jet, Sealing-Wax, Agate, Glaſs and moſt Kinds of Precious Stones attract and repel light Bodies at conſiderable Diſtances.

THE chief Things obſervable in theſe Bodies are. 1. That they don't act, but when heated. 2. That they act more forcibly when heated by rubbing, than by Fire. 3. That, when they are well heated by rubbing, light Bodies will be alternately attracted and repell'd by them, but without any obſervable Regularity whatever. 4. If a Line of ſeveral Yards in length has a Ball, or other Body ſuſpended at one End, and the other End be fixed to a glaſs Tube; when the Tube is heated by rubbing, the Electrical Virtue of the Glaſs will be communicated from the Tube to the Ball, which will attract and repel light Bodies in the ſame Manner, as the Glaſs it ſelf does. 5. If the glaſs Tube be emptied of Air, it loſes its Electricity *.

XII. LASTLY, the Loadſtone is obſerv'd to have Properties peculiar to it ſelf, as that by which it attracts and repels Iron, the Power it communicates to the Needle, and ſeveral others †.

* See *Hauksbee*'s Experiments. *Philoſoph. Tranſact.* No. 326.

† Several Solutions of theſe Properties of *Electricity* and *Magnetiſm* have been attempted by different Philoſophers, but all of them ſo unſatisfactory as not to deſerve a particular Account in this Place, See *Chambers*'s Dictionary in *Electricity*, and *Des Cartes* Opera Philoſophica. P. IV. §. 133. with ſeveral others referr'd to in *Quæſtiones Philoſophicæ*. *Deſagul.* Lect. I. §. 33.

CHAP. IV.

Of the Laws of Motion, commonly called Sir ISAAC NEWTON's *Laws of Nature.*

I. ALL Bodies continue their State of Reſt, or uniform Motion in a right Line, till they are made to change that State by ſome external Force impreſſed upon them.

THIS Law is no other, than that univerſal Property of Bodies, called Paſſiveneſs or Inactivity; whereby they endeavour to continue the State they are in, whatever it be. Thus a Top only ceaſes to run round on Account of the Reſiſtance it meets with from the Air, and the Friction of the Plane whereon it moves. And a Pendulum, when left to vibrate in *vacuo*, where there is nothing to ſtop it, but the Friction ariſing from the Motion of the Pin on which it is ſuſpended, continues to move much longer, than one in the open Air.

II. THE change of Motion, produc'd in any Body, is always proportionable to the Force, whereby it is effected; and in the ſame Direction, wherein that Force acts.

THIS is an immediate Conſequence of this Axiom, the Effect is always proportionable to its

its Cauſe. For Inſtance, if a certain Force produces a certain Motion, a double Force will produce double the Motion; a triple Force triple the Motion *&c.* If a Body is in Motion, and has a new Force impreſſed on it in the Direction wherein it moves, it will receive an Addition to its Motion, proportional to the Force impreſſed; but if the Force acts directly contrary to its Motion, the Body will then loſe a proportional Part of its Motion: again, if the Force is impreſſed obliquely, it will produce a new Direction in the Motion of the Body, more or leſs different from the Former in Proportion to its Quantity and Direction *.

* This Caſe is expreſſed more accurately by Mathematicians thus. If the Proportion and Direction of two Forces, acting upon a Body at the ſame Time, be repreſented by the Sides of a Parallelogram, the Diagonal of that Parallelogram will repreſent the Proportion and Direction of their united Forces.

Dem. Let the Body *A* (*Fig.* 3.) be impell'd with a Force, which would carry it to *E*, in the ſame Time that another, acting upon it in the Direction *AD*, would carry it to *D*. Imagine that while the Body paſſes to *E*, the Line *AD* (in which the Body moves by the other Force) moves to *EB*, in a Direction parallel to it ſelf; when the Body has advanc'd to *G* in the Line *AE*, the Line *AD* will have got to *GF*, and the Body will have paſſed over *GH*, ſuch a Part of it, as bears the ſame Proportion to the whole Line *GF*, as *AG* does to *AE*, that is *GH* (the ſhorter Side of the Parallelogram *GM*, is to *GF*, or, which is the ſame Thing, to *EB* (the ſhorter Side of the Parallelogram *ED*,) as *AG* (the longer Side of the former) is to *AE* (the longer Side of the latter,) from whence the Parallelograms are ſimilar, *El.* 6. *Def.* 1. and conſequently, by 24. *El.* 6. the Point *H* is in the Diagonal, that is, the Body will always be found in the Line *AB*. Q. E. D.

Coroll.

III. Reaction is always contrary, and equal to Action; or the Actions of two Bodies upon each other, are equal, and in contrary Directions.

Thus, suppose a Stone, or other Load to be drawn by an Horse; the Load reacts upon the Horse, as much as the Horse acts upon the Load; for, the Harness, which is stretch'd equally between them both Ways, draws the Horse towards the Stone, as much as it does the Stone towards the Horse; and the progressive Motion of the Horse is as much retarded by the Load, as the Motion of the Load is promoted by the Endeavour of the Horse *. This will be better explain'd from the following Instance; let a Person, sitting in a Boat,

Caroll. From hence we have an easy Method of resolving a given Motion into any two, or more Directions whatever; *viz.* by describing a Parallelogram about the given Direction as a Diagonal, the two Sides of which will represent the Directions sought. Thus, suppose a Body was impell'd in the Line *AB*, we may conceive it as acted upon by two Forces at the same Time, one towards *E*, the other towards *D*, or any other two whatever, provided the Lines be drawn of such length, that, when the Parallelogram is compleated, the given Line *AB* shall be its Diagonal.

* It may be thought perhaps, that (two equal and contrary Forces destroying one another) the Horse will in this Case not be able to move at all, because the Load draws him back, as much as he draws the Load forwards. But it is to be observ'd that the Strength of the Horse is not properly exerted upon the Load but upon the Ground; consequently the Ground reacting and continuing at Rest, pushes the Horse forward with just so much Force as the Horse exerts, above what is counteracted by the Load.

draw

Part I. Plate I. Pag. 22.

Fig. 1. P. 8.

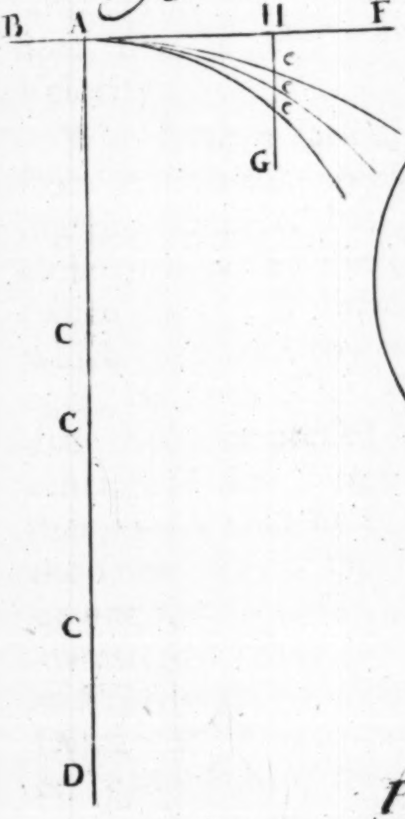


Fig. 2. P. 16.

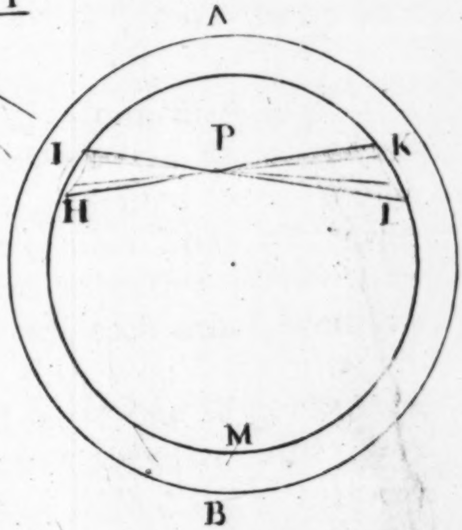
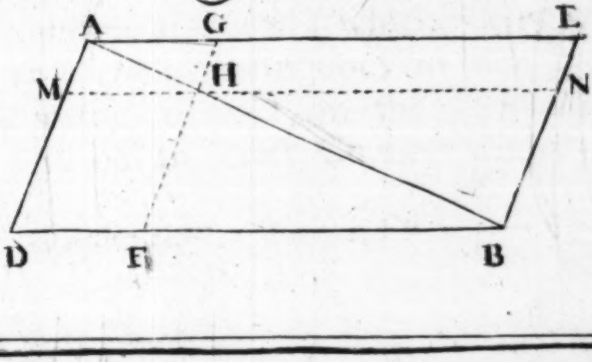


Fig. 3. P. 21.



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draw another Boat equally heavy towards him, they will both move towards each other with equal Velocities: let the Boat he ſits in be the lighteſt, and it will move the faſteſt; becauſe the Action being equal on both Sides, the ſame Quantity of Motion will be given to each Boat, that is, the leſſer will have the greater Velocity *.

We have a farther Confirmation of this from Attraction. Suppoſe two Bodies attracting one another, but prevented from coming cloſe together by ſome other Body placed between them: if their reſpective Actions, by which they tend towards each other, were not equal on both Sides, then would the intermediate Body be preſſed more one Way than the other, and that therefore together with the other two would by this Means begin to move the ſame Way; but that three Bodies ſhould be put into Motion after this Manner, when no external Force acts upon them, is contrary to Experience, conſequently whatever different Degrees of Force, any two Bodies would exert upon others, their mutual Actions on each other are always the ſame. This may be try'd with a Loadſtone and Iron; which, being put into proper Veſſels, contiguous to one another, and made to float on the Surface of Water, will be an exact Counterbalance to

* See the Diſtinction between Motion and Velocity. Chap. 9.

each other, and remain at Reſt, whatever be the attractive Power of the Loadſtone, or the Proportion of their reſpective Magnitudes.

These Laws receive an abundant Additional Proof from hence, *viz.* that all the Concluſions that are drawn from them, in Relation to the Phænomena of Bodies, how complicated ſoever their Motions be, are always found to agree perfectly with Obſervation. The Truth of which ſufficiently appears in all Parts of the *Newtonian* Philoſophy *.

CHAP. V.

The Phænomena of Falling Bodies.

I. THE Laws of Nature being thus explained, we proceed to account for thoſe Phænomena, which are ſolvable by them.

II. To begin with thoſe of Falling Bodies. Conſtant Experience ſhews, that Bodies have a Tendency towards the Earth, which is call'd Gravity, the Laws of which were enumerated in Chap. 3. §. 7.

III. The Height, Bodies can be let fall from, bears ſo ſmall a Proportion to their Diſtance from the Center of the Earth, that it cannot

* See theſe Laws explain'd more at large by *Cheyne* in his Principles of Philoſophy. *Keil's* Introd. ad Phyſ. Præl. 11, 12.

ſenſibly

ſenſibly alter their Gravity; which therefore may be conceiv'd, as acting conſtantly and uniformly upon them, during the whole Time of their fall: from whence they muſt neceſſarily acquire at every Inſtant, an equal Degree of Velocity, which on that Account will conſtantly increaſe, in Proportion to the Time the Body takes up in falling.

IV. THE Spaces Bodies fall through in different Times, reckoning from the Beginning of their fall, are as the Squares of thoſe Times; thus, a Body will fall four Times as far in two Minutes, as it does in one, and nine Times as far in three, ſixteen Times as far in four *&c.**

* In order to demonſtrate this Propoſition, it will be neceſſary to lay down the following Theorem. *viz.*

That the Space a Body paſſes over, with an uniform Motion, is in a Ratio compounded of the Time and Velocity. For the longer a Body continues to move uniformly, the more Space it moves over; and the faſter it moves during any interval of Time, the farther it goes; therefore the Space is in a Ratio compounded of both, that is, is had by multiplying one into the other.

Coroll Therefore the Area of a Rectangle, one of whoſe Sides repreſents the Celerity a Body moves with, and the other the Time of its Motion, will expreſs the Space it moves through.

This being premiſed, let the Line *AB* (*Fig.* 4.) repreſent the Time a Body takes up in falling, and let *BC* expreſs the Celerity acquir'd by its fall; farther let the Line *AB* be divided into an indefinite Number of ſmall Portions, *ei*, *im*, *mp*, &c, and let *ef*, *ik*, *mn*, *pq*, &c. be drawn parallel to the Baſe. Now it is evident from §. 3. (*viz.* that the Velocities are as the Times in which they are acquir'd) that the Lines *ef*, *ik*, *mn*, *pq*, &c. being to each other (4. *El.* 6) as the Lines *Ae*, *Ai*, *Am*, *Ap*, &c. will repreſent the Celerities in the Times repreſented by theſe: that is, *ef* will be as the Velocity of the

V. From this Propoſition it follows, that a Body falls three Times as far, in the ſecond Portion of Time, as it does in the firſt; five Times as far in the third; ſeven Times in the fourth, and ſo on in the Series of the odd Numbers: for otherwiſe, it could not fall four Spaces in two Minutes, and nine in three, as the Propoſition aſſerts *.

Body in the ſmall Portion of Time *ei*, and *ik* will be as the Velocity in the Portion of Time *im*; in like Manner *pq* will be as the Velocity in the Portion of Time *po*, which Portions of Time being taken infinitely ſmall, the Velocity of the Body may be ſuppos'd the ſame, during any whole Portion; and conſequently, by the Corollary of the foregoing Theorem, the Space run over in the Time *ei* with the Velocity *ef* may be repreſented by the Rectangle *if*: in like Manner the Space run over in the Time *im* with the Celerity *ik*, may be expreſs'd by the Rectangle *mk*; and that run over with the Celerity *mn* in the Time *mp*, by the Rectangle *pn*; and ſo of the reſt. Therefore the Space run over in all thoſe Times will be repreſented by the Sum of all the Rectangles, that is, by the Triangle *ABC*, for thoſe little triangular Deficiencies, at the End of each Rectangle, would have vaniſhed, had the Lines *ei*, *im*, *mp*, &c. been infinitely ſhort, as the Times they were ſuppoſed to repreſent. Now as the Space, the Body deſcribes in the Time *AB*, is repreſented by the Triangle *ABC*, for the ſame Reaſon the Space paſs'd over in the Time *Ao* may be repreſented by the Triangle *Aor*, but theſe Triangles being ſimilar, are to each other, as the Squares of their homologous Sides *AB* and *Ao* (20. *El.* 6.): that is, the Spaces repreſented by the Triangles are to each other, as the Squares of the Times repreſented by the Sides. *Q. E. D.*

* This may alſo be ſhewn in the following Manner. Let the Triangle *ABC* (*Fig.* 4.) be divided into leſſer ones, as in *Fig.* 5. each equal to *Dbr*, which repreſents the Space deſcribed by the falling Body in *Db* the firſt Portion of Time; 'tis evident that in *bc* the ſecond Portion of Time, there are three ſuch Triangles deſcribed, *viz.* thoſe that lie between the Lines

VI. THE Spaces describ'd by Falling Bodies in different Times, are as the Squares of the last acquir'd Velocities. For by §. 4. the Spaces are as the Squares of the Times, and by §. 3. the Velocities are as the Times; therefore the Spaces are also as the Squares of the Velocities.

VII. THE Space a Body passes over from the Beginning of its fall in any determinate Time, is half what it would describe in the same Time moving uniformly with its last acquir'd Velocity *.

VIII. IN like Manner, when Bodies are thrown up perpendicularly, their Velocities decrease, as the Times they ascend in increase; their Gravity destroying an equal Portion of their Velocity every Instant of their Ascent.

IX. THE Heights Bodies rise to, when thrown perpendicularly upwards, are as the Squares of the Times spent from their first setting out, to the Moment they cease to rise. That is, if a Body is thrown with such a Degree of Velocity, as to continue rising twice as

Lines *br* and *cs*; in *cd* the third Portion of Time, five such, *viz.* all between *cs* and *dt*; in *df* the next equal Portion of Time, seven such, &c.

* For let the Time be *AB*, (*Fig.* 4.) and the last Velocity *BC*, the Space the Body runs over, while it is acquiring that Velocity, is *ABC*, but the Space it would pass over in the Time *AB*, was it to move uniformly with the Celerity *BC*, is by the Theorem, (Note p. 25.) the Space *ABCD*, double the Former. *Q. E. D.*

long as another, it shall ascend four Times as high; if thrice, nine Times as high, *&c.*

THESE two are the converse of the third and fourth Sections *.

CHAP. VI.

Of the descent of Bodies on oblique Planes, and of Pendulums.

WHEN a Body descends on an oblique Plane, its Motion is continually accelerated by the Action of Gravity, but in a less Degree, than when it descends perpendicularly; its free Descent in this Case being hinder'd by the Interposition of the Plane: from whence it follows, that what was said in the last Chapter, concerning the perpendicular Descent of Bodies, is true of such as fall on oblique Planes, Allowance being made for the difference of Acceleration.

II. THE Effect Gravity has upon a Body falling down an oblique Plane, is to that which it exerts upon another falling freely; as the perpendicular Height of the Plane is to its Length †.

* See *Keil's* Introd. ad Phys. Præl. 11. *Gravesande* L. 1. Ch. 17.

† *Dem.* Let *AC* (*Fig.* 6.) be the inclin'd Plane, the Body at *A*, and the Action of Gravity, whereby it endeavours to fall per-

III. The Space, through which a Body falls down the oblique Side of a Plane, is to that through which it would fall perpendicularly in the ſame Time; as the perpendicular Height of the Plane is to its Length *.

For the Space, a Body falls through in any determinate Time, whether down an inclined Plane, or not, is as the Effect of the Gravity with which it is acted upon during that Time; but the Gravity, with which a Body deſcends down the oblique Side of a Plane, (by the laſt Propoſition) is to that with which it falls perpendicularly, as the perpendicular Height of the Plane to its Length: the Space therefore, which a Body falls through obliquely, is to that which it would paſs through perpendicularly in the ſame Time, alſo in that Proportion.

perpendicularly, repreſented by the Line *AB*; let *AD* be perpendicular to *AC*, *AD* will then repreſent the Direction by which the Plane acts upon the Body (for all Bodies act in Lines perpendicular to their Surfaces,) let then thoſe two Forces be reſolved into one in the Direction *AC*, (as ſhewn in Note to §. 4. Chap. 4.) by compleating the Parallelogram *BD* whoſe Diagonal will be *AG*. In order to this *BG* muſt be let fall perpendicularly upon *AC* (that it may be parallel to the oppoſite Side of the Parallelogram *AD*) conſequently (8. *Elem.* 6.) *AG* is to *AB* as *AB* to *AC*, that is, the Tendency of the Body down the Plane is to its perpendicular Tendency, as *AB* is to *AC*. Q. E. D.

* From this Propoſition it follows, that ſuppoſing *BG* (*Fig.* 6.) perpendicular to *AC*, the Body would fall from *A* to *G*, in the ſame Time another would fall from thence to *B*; for, as was obſerv'd (Note the laſt) *AG* is to *AB*, as *AB* to *AC*.

IV.

IV. The Velocity a Body acquires by falling perpendicularly, is to that, which it acquires by falling obliquely in the same Time, as the Space of its perpendicular Descent is to that of its oblique one *.

V. The Time, in which a Body descends through the oblique Side of a Plane, is to that in which it falls through the perpendicular Height of the same; as the Length of the oblique Side is to its Height †.

VI. A Body acquires the same Velocity in

* Since by the Note to Section the last, a Body falls to *G*, (*Fig.* 6.) in the same Time another falls to *B*, and by (Chap. 5. §. 7.) the Space a falling Body passes over in any Time, is half that which it would run over in the same Time moving uniformly with its last acquir'd Velocity, it follows that the Body falling down the oblique Plane would pass over double the Space *AG*, moving uniformly with its last acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; likewise double the Space *AB* would be pass'd over by the other Body, moving uniformly with its last acquir'd Velocity, in a Portion of Time equal to that in which it was acquir'd; but since the Velocities of Bodies moving uniformly are as the Spaces they run over in equal Times, the Velocities of the Bodies in *G* and *B* are to each other as double the Lines *AG* and *AB*, that is as the Lines themselves, which by §. 3. are as the Spaces run through in the same Time, from whence the Proposition is clear.

† *Dem.* The Square of the Time in which *AC* (*Fig.* 6.) is run over, is to the Square of the Time in which *AG* is run over, as *AC* to *AG*, (by Chap. 5. §. 4.) that is, since *AC*, *AB*, *AG* are continually proportional (8. *Elem.* 6.) as the Square of *AC* to the Square of *AB* (by *Def.* 10. *Elem.* 5.) therefore the Times themselves are as the Lines *AC* and *AB*, that is, as the oblique Side of the Plane to the perpendicular Height. *Q. E. D.*

falling

falling down the oblique Side of a Plane, as it would do, if it fell freely through the perpendicular Height of it *.

VII. A Body takes up the ſame Time in falling through the Chord of a Circle, whether it be long or ſhort, as it does in falling perpendicularly through the Diameter of the ſame Circle †.

VIII. UPON this is founded the Theory of Pendulums: for from hence it follows, that ſuppoſing a Pendulum could be made to vibrate in a Chord of a Circle, inſtead of an Arch, all its Vibrations would require the ſame Time, whether they were large or ſmall ‡.

* *Dem.* The Square of the Velocity a Body acquires by falling to *G*, is to the Square of the Velocity it acquires by falling to *C*, as the Space *AG* to the Space *AC* (by Chap. 5. §. 4.) that is (by 8. *Elem.* 6. and *Def.* 10. *Elem* 5.) as *AGq* to *ABq*. But ſince *AG* is run over in the ſame Time *AB* is, (ſee Note to §. 3.) the Velocity in *G* is to the Velocity in *B*, as *AG* to *AB*, (by §. 4.) and conſequently ſince the Velocities both in *C* and *B* bear the ſame Proportion to that in *G*, they muſt be equal to each other. *Q. E. D.*

† *Dem.* It was demonſtrated (§. 3.) that a Body will fall from *A* to *G*, (*Fig.* 7.) on the inclin'd Plane *AC*, in the ſame Time another would fall freely to *B*, provided *AGB* is a right Angle, in which Caſe *AG* (by 31. *Elem.* 3.) is a Chord of that Circle of which *AB* is the Diameter; therefore a Body falls through the Chord &c. Q. E. D.

‡ This may be illuſtrated by conceiving the laſt Figure inverted (as in *Fig.* 8.) where ſuppoſing the Ball ſuſpended in ſuch a Manner, as to ſwing in the right Line *GA* inſtead of the Arch *GA*, it would always fall through it in the ſame Time, however long or ſhort it was, for the Inclination of the

IX. FROM hence we ſee the Reaſon, why the ſhorter Arches a Pendulum deſcribes, the nearer its Vibrations come to an Equality, for ſmall Arches differ leſs from their Chords than large ones. But if the Pendulum is made to vibrate in a Curve, which Mathematicians call a *Cycloid*; each Swing will then be perform'd in the ſame Time, whether the Pendulum moves through a larger or leſſer Space. For the Nature of this Curve is ſuch, that the Tendency of a Pendulum towards the loweſt Point of it, is always in Proportion to its Diſtance from thence; and conſequently let that Diſtance be more or leſs, it will always be run over by the Pendulum in the ſame Time *.

the Line *GA* to the horizontal Line *BC*, is not alter'd by inverting the Figure.

* The Deſcription of a *Cycloid*,

Upon the right Line *AB*, (*Fig.* 9.) let the Circle *CDE* be ſo plac'd, as to touch the Line in the Point *C*, then let this Circle roll along upon it from *C* to *H*, as a Wheel upon the Ground, then will the Point *C* in one Revolution of the Circle deſcribe the Curve *CKH*, which is call'd a Cycloid. Now ſuppoſe two Plates of Metal bent into the Form *HK* and *KC*, and placed in the Situation *LH* and *LC*, in ſuch Manner, that the Points *H* and *C* may be apply'd to *L*, and the Points anſwering to *K* be apply'd to *H* and *C*. This done, if a Pendulum as *LP*, in length equal to *LH*, be made to vibrate between the Plates or Cheeks of the Cycloid *LC* and *LH*, it will ſwing in the Line *CKH*; and the Time of each Vibration, whether the Pendulum ſwings through a ſmall or a great Part of the Cycloid, will be to the Time a Body takes up in falling perpendicularly through a Space equal to *IK*, (half the length of the Pendulum;) as the Circumference of a Circle to its Diameter, and conſequently it will always be the ſame.

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Fig. 4. P. 25.

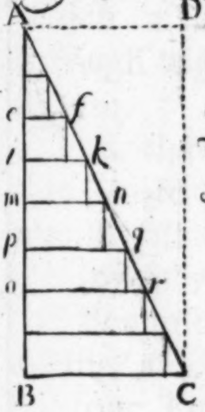


Fig. 5. P. 26.

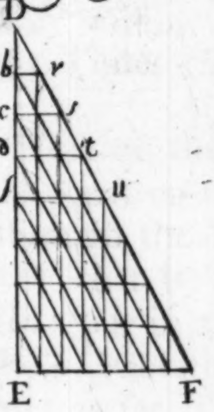


Fig. 6. P. 28.

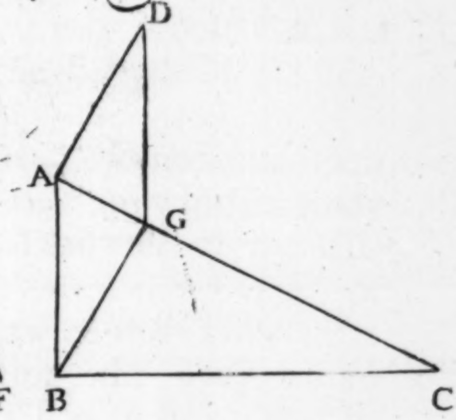


Fig. 7. P. 31.

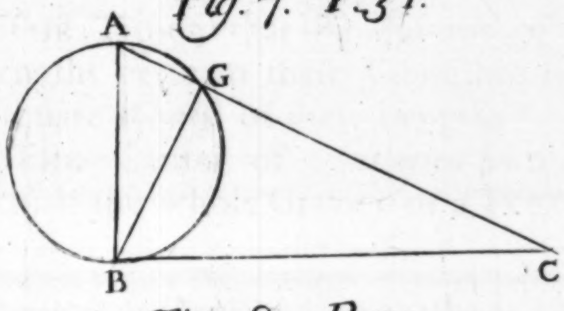
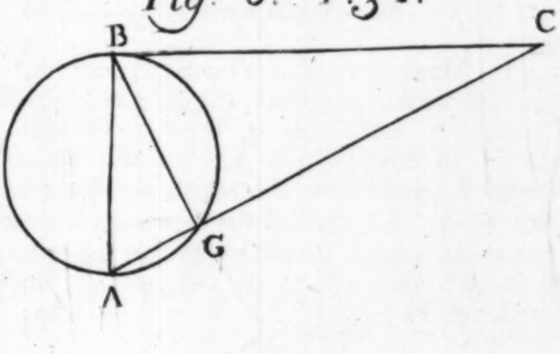


Fig. 8. P. 31.



X. THE Time of the Deſcent and Aſcent of a Pendulum, ſuppoſing it to vibrate in the Chord of a Circle, is equal to the Time in which a Body falling freely would deſcend through eight Times the Length of the Pendulum.

FOR the Time of the Deſcent alone upon the Chord is equal to that in which a Body would fall through the Diameter of the Circle (by §. 7.); that is, twice the Length of the Pendulum: but in twice that Time (*viz.* during a whole Vibration) the Body would fall four Times as far (Chap. 5. §. 4.), that is, through eight Times the Length of the Pendulum.

XI. THE Times, that Pendulums of different Lengths perform their Vibrations in, are as the ſquare Roots of their Lengths *.

XII. THE Center of *Oſcillation* is a Point in which, if the whole Gravity of a Pendulum

They that would ſee a Demonſtration of this and ſeveral other Things relating to this Curve, may conſult *Huygens Horol. Oſcillatorium*, or *Cotes*'s *Harmonia Menſurarum.*

* *Dem.* Let there be two Pendulums *A* and *B* (*Fig.* 16. and 11.) of different Lengths, the Time the firſt vibrates in (ſuppoſe through a Chord) is equal to the Time in which a Body would fall freely through *DA*, the Diameter of the Circle (as demonſtrated §. 7.); in like Manner the Time *B* vibrates in, is that in which a Body would fall through *FB*. Now the Times in which Bodies fall through different Spaces are as the Square Roots of thoſe Spaces, that is, of *DA* and *FB*, or of their Halves *CA* and *CB*, *i. e.* of the Lengths of the Pendulums. *Q. E. D.*

E was

was collected, the Time of its Vibration, would not be alter'd thereby *; this is the Point from whence the Length of a Pendulum is meaſur'd, which in our Latitude, in a Pendulum that ſwings Seconds, is thirty nine Inches and two Tenths.

XIII. THE Squares of the Times in which Pendulums, acted upon by different Degrees of Gravity, perform their Vibrations in, are to each other, inverſely as the Gravities †.

* The Rule for finding the Center of Oſcillation.

If the Ball *AB* (*Fig.* 12.) be hung by the String *CD*, whoſe Weight is inconſiderable, the Center of Oſcillation is found thus; ſuppoſe *E* the Center of the the Globe, take the Line *K* of ſuch a Length, that it ſhall bear the ſame Proportion to *ED* as *ED* to *EC*, then *EH* being made equal to $\frac{2}{5}$ of *K*, the Point *H* ſhall be the Center of Oſcillation.

If the Weight of the Rod *CD* be too conſiderable to be neglected divide *CD* (*Fig.* 13.) in *I*, ſo that *DI* may be equal to $\frac{1}{3}$ of *CD*, and make a Line as *G*, in the ſame Proportion to *CI*, that the Weight of the Rod bears to that of the Globe, then having found *H* the Center of Oſcillation of the Globe, as before, divide *IH* in *L*, ſo that *IL* may bear the ſame Proportion to *LH*, as the Line *CH* bears to the Line *G*; then will *L* be the Center of Oſcillation of the whole Pendulum. See *Huygens* Horol. Oſcillat.

† *Dem.* The Spaces falling Bodies deſcend through, are as the Squares of the Times, when the Gravity by which they are impell'd is given (Chap. 5. §. 4); and as the Gravity when the Time is given (for the Sum of the Velocities produced in any Time will always be as the generating Forces): conſequently when neither is given, they are in a Ratio compounded of both; the Squares of the Times are therefore inverſly as the Gravities. [*For if in* 3 *Quantities* a, b, c; a *is as* bc, *then* b : $\frac{a}{c}$, i. e. *if* a *is given, as* $\frac{1}{c}$ *or as* c *inverſly.*] But if

the

From whence it follows, that a Pendulum will vibrate slower when nearer the Equator, than the same when nearer the Poles; for the Gravity of all Bodies is less, the nearer they are to the Equator; *viz.* on account of the spheroidical Figure of the Earth, and its Rotation about its Axis, as will be explain'd hereafter. To which we may add the Increase of the Length of the Pendulum occasion'd by the Heat in those Parts; (for we find by Experiment that Bodies are inlarged in every Dimension, in Proportion to the Degree of Heat that is given them:) for which Reason (§. 11.) the Vibrations of the Pendulum will also be slower.

CHAP. VII.

Of Projectiles.

A BODY projected in a Direction parallel or oblique to the Horizon, would proceed on in infinitum in a right line (by the first Law of Nature); but being continually

the Squares of the Times in which Bodies fall through given Spaces are inversly as the Gravities by which they are acted upon; then the Squares of the Times in which Pendulums of equal Lengths, perform their Vibrations, will be also in the same Ratio, on account of the constant Equality between the Time of the Vibration of a Pendulum, and of the descent of a Body through eight Times its Length (§. 12.)

accelerated towards the Earth by its Gravity, it will describe a Curve called a *Parabola* *.

* *Dem.* Let us suppose the Body thrown from *A* in the Direction *AB* horizontally (*Fig.* 14.) or obliquely (*Fig.* 15.) it would (if not attracted towards the Earth) move uniformly from *A* towards *B*, that is, in equal Times it would describe equal Parts of the Line *AB*, as *AC, CD, DE*, &c. but, if in the first Portion of Time, while it would move from *A* to *C*, it would have descended from *A* to *G* by its Gravity, had it only been let drop from thence; it will, by a Composition of these two Motions (Chap. 4. §. 2.) at the End of that Time be found in *H*, the opposite Angle of the Parallelogram *ACGH*. Then in twice that Time, *viz.* while it would have moved over two equal Portions, or from *A* to *D*, it would fall downwards to *M*, four Times as far as before (Chap. 5. §. 4.) and will therefore be found in *I*, supposing *DI* equal and parallel to *AM*. Then again in three Portions of Time, or while it would have moved over three Divisions, that is, from *A* to *E*, it would have fallen downwards nine Times as far as in the first Portion of Time, and therefore being carried by these two Motions will at the End of that Time be found in *K*, supposing *EK*, or its equal *AN*, nine Times as long as *AG* or *CH*, &c. Therefore the Lines *CH, DI, EK*, &c. which are to each other as the Numbers 1, 4, 9, &c. are as the Squares of the Lines *AC, AD, AE*, (these being only as the Numbers 1, 2, 3.) But this is the Property of the Parabolic Curve, (See De L' *Hospital* B. I Prop. 1. Corol. 2. and Prop. 3. Corol. I.) Consequently the Curve *AHIK* &c. which the Body moves in, whether thrown horizontally or obliquely, is a Parabola. *Q. E. D.*

Def. The Quotient which arises from the Division of the Square of the Line *AC* by the Line *AG*, *viz.* the Quantiy $\frac{ACq}{AG}$ (in either of the Parabolic Curves, *Fig.* 14. or 15.) is called the *Parameter* of that Point: and the Square of the Line *AD* divided by *AM*, viz. $\frac{ADq}{AM}$, or the Square of *AE* divided by *AN*, viz. $\frac{AEq}{AN}$ being demonstrated by the Writers on Conic Sections to be equal to it, any of these Quantities are indifferently put to express the Parameter of the same Point.

Lemma.

Part. I. Plate III. *Pag.* 36

Fig. 9. *P.* 32.

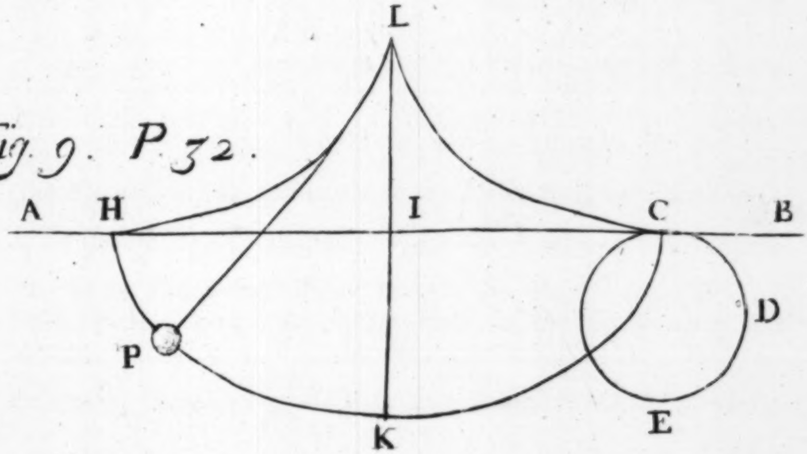


Fig. 10. *P.* 33.

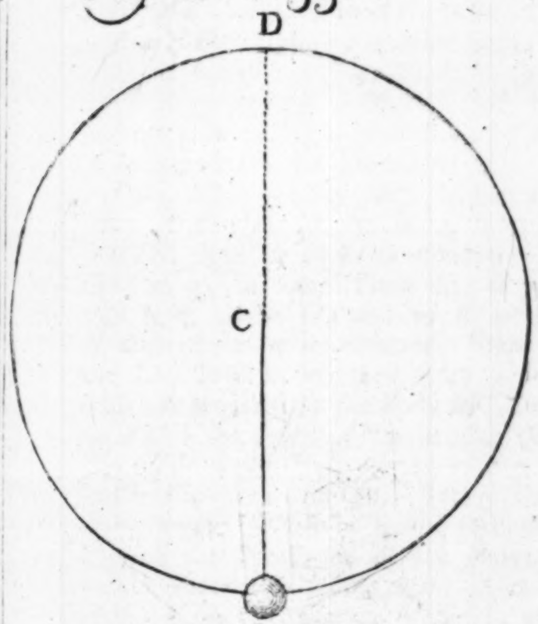
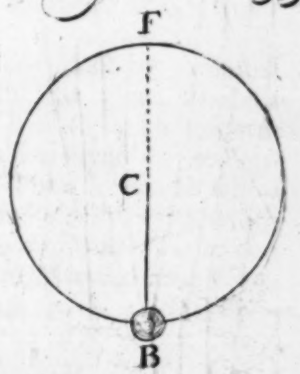


Fig. 11. *P.* 33



Lemma. The Velocity a Body would acquire by falling from an Height equal to the fourth Part of the Parameter of the Point A, is to the Velocity it would acquire by falling from A to N, as AE is to twice AN.

Dem. Since we are comparing the Velocity, which a Body would acquire by falling through a fourth Part of the Parameter, with that which it would acquire by falling to N, let $\frac{AEq}{AN}$ (the Expression last mention'd in the Definition) be made choice of to denote the Parameter. Then $\frac{\frac{1}{4}AEq}{AN}$ will express a fourth Part of the Parameter. Now because the Velocities acquir'd by falling Bodies, are as the square Roots of the Spaces they fall through (Chap. 5. §. 6.), the Velocity acquir'd by a Body in descending through $\frac{\frac{1}{4}AEq}{AN}$, is to that Velocity, which it would acquire by falling through AN, as the square Root of $\frac{\frac{1}{4}AEq}{AN}$ to the square Root of AN; that is, extracting the Roots of those Quantities, as $\frac{\frac{1}{2}AE}{\surd AN}$ to $\surd AN$, and, multiplying each Term by $\surd AN$. as $\frac{1}{2}AE$ to AN, or as AE to twice AN. Q. E. D.

Prop. The Velocity a Body ought to be projected with, to make it describe a given Parabola, is such as it would acquire by falling through a Space equal to the fourth Part of the Parameter belonging to that Point of the Parabola, from whence it is intended to be projected.

Dem. The Velocity with which a Body must be projected from A towards B, to make it describe the given Parabola $AHIK$, must be such, as would carry it to C by an uniform Motion, in the same Time that it would descend by its Gravity from A to G; and to E in the Time it would fall to N &c. as was before observed. Now the Velocity, with which the Line AE is described with an uniform Motion, is to that which is acquired by the Body in falling to N in the same Time, as AE is to twice AN; because (Chap. 5. §. 7.) its Velocity in N would have carried it over twice AN in that Time, had it also been uniform. But by the Lemma, the Velocity a Body would acquire by falling through a Space equal to a fourth Part of the Parameter of the Point A, is to that which it would acquire by falling from A to N, also as AE to twice AN. Since therefore the Velocity, with which the Line AE

is

s described, (or, which is the same Thing, that whereby the Body is projected,) and that which a Body would acquire by falling through a fourth Part of the Parameter of the Point *A*, bear one and the same Proportion to that Velocity which a Body would acquire by falling from *A* to *N*, they must be equal. *Q. E. D.*

Corol. This affords us an easy Method of finding what Direction it is necessary to throw a Ball in with a given Velocity, in order to strike an Object in a given Situation. *v.g.* Let it be requir'd to strike an Object as *K*, with a Ball thrown from *A* with a given Velocity. Here it is only necessary to make the Triangle *ANK* (suppose a right Line drawn from *A* to *K*) such, that $\frac{NKq}{AN}$ or which is the same Thing $\frac{AEq}{EK}$ in the Triangle *AEK*, may be equal to four Times the Space a Body must fall through, to acquire such a Degree of Velocity as that with which it is intended to be thrown, and then *AE* will be the Direction sought. In order to this we must lay down the following Lemma.

Let there be a Circle as *ABC* (*Fig.* 16.) *AK* a Tangent in the Point *A*, *AB* and *KI* parallel to each other, and let the other Lines be drawn, as in the Figure, I say $\frac{AEq}{EK} = AB$. For the Angle *ABE* is equal to the Angle *EAK* (32. *Elem.* 3.) and the Angle *BAE* is equal to the Angle *AEK* as alternate, therefore the Triangles *ABE* and *AEK* are similar; consequently *AB* is to *AE*, as *AE* to *EK*, and multiplying the extreme Terms together, and middle Terms together, $AB \times EK = AEq$ and dividing both Sides of the Equation by *EK*, $AB = \frac{AEq}{EK}$. Q. E. D. By the same Method of arguing $\frac{AIq}{IK}$ may be proved equal to *AB*.

The Problem.

Let it be requir'd to strike an Object as *K* (*Fig.* 17.) with a Ball projected from *A* with a given Velocity.

Solution. Erect *AB* perpendicular to the Horizon, and equal to four Times the Height a Body must fall from, to acquire the Velocity with which the Ball is to be thrown; bisect this in the Point *G*, through which draw *HC* perpendicular to *AB*, and meeting the Line *AC* (perpendicular to

II. THE greateſt horizontal Diſtance to which a Body can be thrown with a given Velocity, is at the Elevarion of 45 Degrees *.

III. IF two Balls are thrown at different Elevations (but with equal Degrees of Velocity,) the one as much above forty five Degrees as the other below, the horizontal Diſtances (or Randoms) where they both fall, will be the ſame †.

AK) in *C*. On *C* as a Center with the Radius *CA*, deſcribe the Circle *ABD*; laſtly through *K* draw the Line *KEI* perpendicular to the Horizon, cutting the Circle in the Points *E* and *I*; I ſay *AE* or *AI* will be the Direction ſought.

For by the Lemma, $AB = \frac{AEq}{EK}$ or $\frac{AIq}{IK}$, but *(ex conſtructione) AB* is equal to four Times the Height a Body muſt fall from, to acquire the Velocity with which it is to be thrown, therefore its equal $\frac{AEq}{EK}$ or $\frac{AIq}{IK}$, is the ſame, which by the Corollary was the Thing requir'd to determine the Direction ſought; conſequently the Parabola, which the Body will deſcribe, will paſs through the Point *K*. Q. E. D.

Coroll. 1. From hence it is evident, that if the Object to be ſtruck, be placed any where in the horizontal Line *AO (Fig.* 18*)* beyond *Q*, the Problem is impoſſible; for then *QH* will not touch the Circle, and the Ball will not reach that Point with any Direction whatever;

* And that when the Ball is directed towards *H*, it will fall on *Q* the greateſt Diſtance it can poſſibly be thrown to; but the Angle *QAH* being equal to *ABH* in the oppoſite Segment *(*32. *Elem.* 3.*)* is equal to half *AGH* at the Center *(*20. *Elem.* 3.*)* which is a right one; conſequently *QAH* is an Angle of 45 Degrees.

† *Coroll.* 2. If the Object is ſituated in the horizontal Line *AO (Fig.* 19.*)* but nearer to *A*, than the greateſt horizontal Diſtance at which it may be ſtruck, ſuppoſe in *K*; the two Directions

IV. The Height a Body will riſe to, when thrown perpendicularly upwards, is equal to half the greateſt horizontal Diſtance it can be thrown to, with the ſame Velocity *.

From hence we may eaſily know how far a Mortar-Piece, or other ſuch Machine, will carry a Ball. Let the Ball be ſhot perpendicularly upwards, note the Time of its Aſcent and Deſcent, half that is the Time of Deſcent, from whence we learn the Height, from which it falls, (for Bodies are obſerv'd to fall in the firſt Second of Time ſixteen Feet, conſequently in two Seconds they fall four Times ſixteen Feet (Chap. 5. §. 4.) in three nine Times as much *&c.*) but the perpendicular Height from whence it falls is the ſame with that to which it aſcended, conſequently (§. 4.) the double of this is equal to the greateſt horizontal Diſtance to which that Machine will carry the Ball with an equal Charge.

Directions *AE* and *AI* with which it may be hit, are equally diſtant from the Direction *AH*; for the Angles *IAH* and *HAE* are equal, as inſiſting on equal Arches *IH* and *HE* (28. *Elem* 3.)

* *Coroll.* 3. The Altitude of a perpendicular Projection is equal to a fourth Part of the Height *AB*; for the Velocity with which the Body is projected, is (*ex hypoth.*) ſuch as it would acquire by falling through a fourth Part of the Line *AB*; but a fourth Part of the Line *AB* is equal to half the Line *GH*, or *AQ* (*Fig.* 18.) that is, half the greateſt horizontal Diſtance to which the Body can be thrown.

See *Cotes*'s Harmonia Menſurarum p. 87. *Keil*'s Introduct. ad Phyſ. Præl. 16.

V. The

Fig. 12. P. 34.

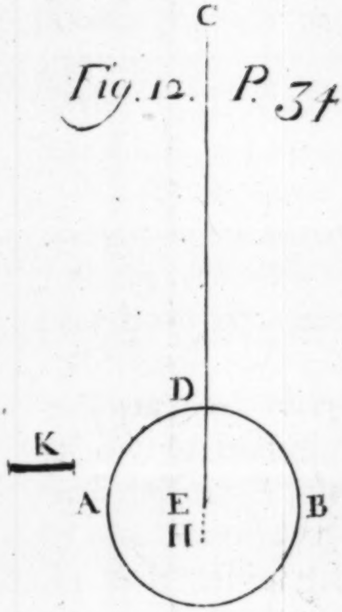


Fig. 13. P. 34.

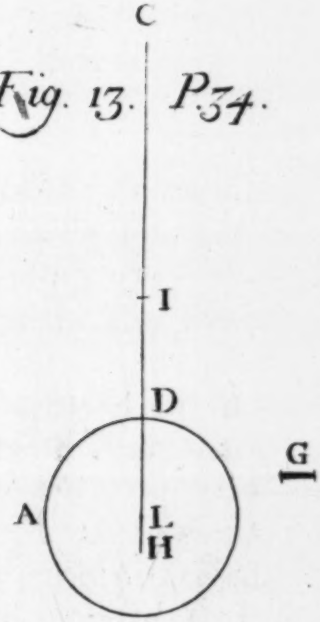
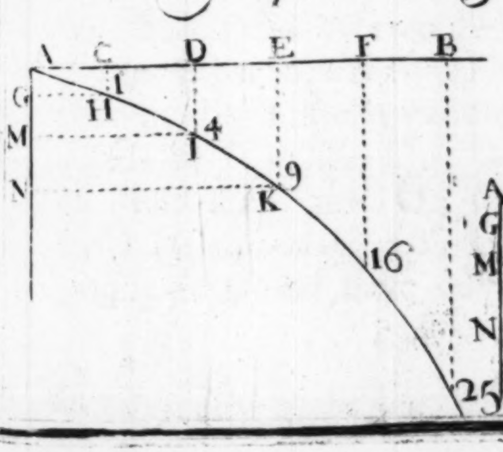
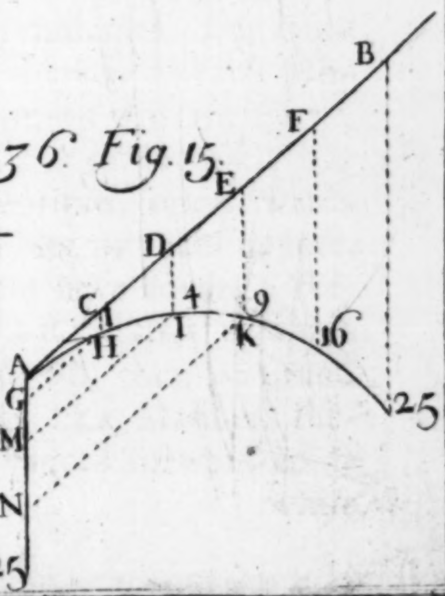


Fig. 14.



P. 36. Fig. 15.



V. THE Randoms of two Projectiles, having the ſame Degrees of Elevation, but thrown with different Velocities, are as the Squares of the Velocities: for by the laſt, the Randoms are equal to double the Heights to which the Bodies thrown perpendicularly upwards will aſcend, but the Heights are (Chap. 5. §. 6.) as the Squares of the Velocities, therefore the Randoms are ſo too.

VI. SUPPOSING the Motion of the Earth, all Bodies, when thrown perpendicularly upwards, deſcribe *Parabola's*; notwithſtanding they appear both to aſcend and deſcend in the ſame right Line.

THIS may very eaſily be illuſtrated in the following Manner; let there be a Body carried uniformly along the Line AB (*Fig.* 20.) by the Motion of the Earth from A towards B; as it paſſes the Point C let it be projected upwards, by ſome Force acting underneath it in the Direction CO perpendicular to the former; the Body will not thereby loſe its Motion, which it had in common with the Earth, towards B (by the firſt Law of Nature), but will be carried by two Motions, one towards B the other towards O; let us then ſuppoſe, that in the Time it would have advanc'd forwards to P in the Line AB, it riſes upwards to M in the Line CO; it will then be found in D (Chap. 4. §. 2.): in like Manner, ſuppoſing it would have advanced forward to Q

F while

while it riſes to N, it would then be found in E, afterwards in F, then in G *&c*: deſcribing the Curve CGL which (from what was demonſtrated under §. 1.) is a Parabola *.

THE Reaſon, why it appears to a Spectator to riſe and fall perpendicularly, is becauſe he is carried uniformly along with it by the Motion of the Earth in the direction AB. *v. g.* Suppoſe the Spectator at C at the inſtant the Body is thrown from thence, when it arrives at D, he will be moved to P, when the Body is at E he will be at Q *&c.* as is evident from what was obſerved about the Motion of the Body in the Curve; and they will both meet in L. Therefore ſince the Spectator imagines himſelf ſtanding ſtill, and ſees the Body always perpendicularly over his Head, he muſt of Courſe think, that it riſes right up, and falls right down.

IT may be proper to obſerve here, that Experiments relating to the Motion of projected Bodies, do not exactly anſwer the Theory; the Reſiſtance of the Air deſtroying Part of their Motion: for which a ſmall Allowance is to be made.

* *Dem.* Suppoſe the Motion the Body had in common with the Earth towards *B (Fig. 21.)* and that with which it is projected towards *O*, ſuch, as being compounded (Ch. 4. §. 2.) would have produced a Motion in the Direction *CX*; it will follow from thence, that the Path deſcribed by it will be the ſame, as if it had been thrown in that Direction from a Point as *C* at reſt; but in that Caſe it would have deſcrib'd a Parabola as *CGL* (§. 1.) therefore alſo in this. *Q. E. D.*

CHAP.

CHAP. VIII.

Of Centripetal and Centrifugal Forces.

WHEN a Body is projected in an horizontal Direction, and by its Gravity made to describe a Parabola, as demonstrated Chapter the last; the Curvature of that Parabola will vary in Proportion to the Velocity with which the Body is thrown, and the Gravity which impels it towards the Earth. For the less its Gravity is in Proportion to the Quantity of Matter it contains, or the greater the Velocity is with which it is projected; the less will it deviate from a straight Line, and the further it will go, before it falls to the Earth. For Instance, if a Bullet be shot out of a Cannon from the Top of a Mountain with a given Velocity in an horizontal Direction, and goes in a Curve Line, suppose to the Distance of two Miles from the Foot of the Mountain before it falls to the Ground; the same Bullet, shot with a much greater Velocity, would fly to a much greater Distance before its fall. And by encreasing the Velocity, the Distance to which it is projected, may be encreased as much as you please; so that it will not fall to the Ground, till it is arrived at the Distance of ten, or thirty, or ninety

Degrees; or till it has even ſurrounded the whole Earth, and arrives at the very Top of the Mountain from whence it was projected: in this Caſe it will perform a ſecond Revolution, and ſo on *in infinitum* without a new Projection, provided the Reſiſtance of the Air is taken away. Nay it may be projected with ſuch Violence, that it will continually recede from the Earth, moving in a Curve, till at length it gets out of the Sphere of the Earth's Attraction; after which it will go on in a ſtraight Line without ever returning. Which may thus be illuſtrated.

LET ABC (*Fig.* 22.) repreſent the Earth, M the Top of the Mountain from whence the Body is projected in the Direction MQ: it may be thrown with ſuch Force as to carry it to B before it falls, or to C, or even to go round to M, deſcribing the Circle MDM; or laſtly it may be made to deſcribe the Curve MO, till it gets out of the Sphere of the Earth's Attraction, ſuppoſe at O, going on afterwards in the infinite ſtraight Line OX, there being nothing to ſtop or alter its Courſe. Farther it may be projected with ſuch a Force from M (*Fig.* 23.) as will cauſe it continually to recede from the Earth, till it arrives at the oppoſite Point G, deſcribing the Curve MKG; and if the Point G is within the Sphere of the Earth's Attraction, the Body will return to M, deſcribing the Curve GLM exactly ſimilar

Part I. Plate V. *Pag.* 44.

Fig 16 *P.*38.

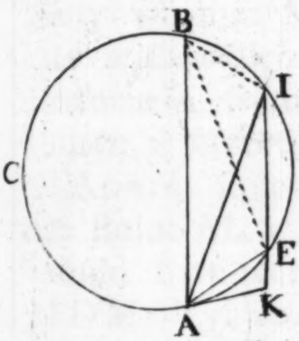


Fig. 17. *P.*38.

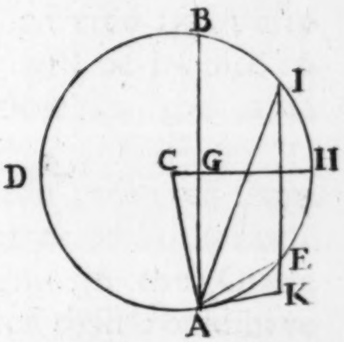


Fig. 18. *P.* 39.

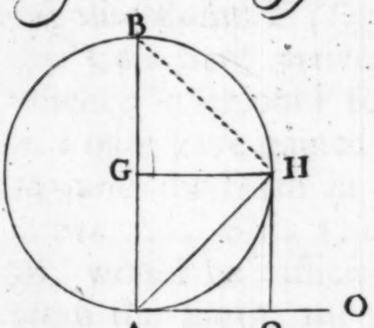


Fig. 19. *P.* 39.

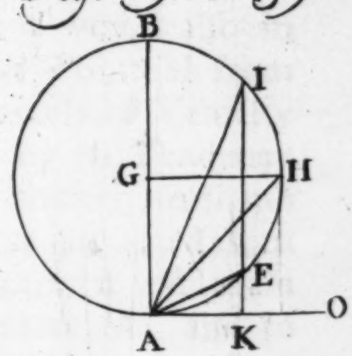


Fig. 20. *P.* 41.

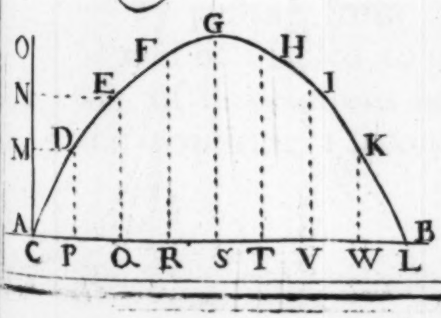
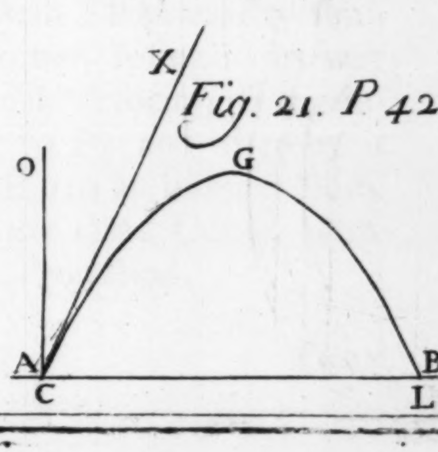


Fig. 21. *P.* 42.



milar to MKG; and in moving nearer and nearer to the Earth till it comes to M, will regain what Velocity it loſt in going from M to G, its Gravity conſpiring with its Motion from G to M in the ſame Degree in which it oppoſed it from M to G; conſequently the Body when at M, having recovered the Velocity with which it ſet out, will be inabled to perform a ſecond Revolution in the ſame Curve as before; and ſo on.

Again, ſuppoſe it had been projected from the Point M, with a leſs Degree of Force than would have carried it round in the Circle MDM *(Fig. 22.)*, but greater than would have ſuffered it to have fallen to the Earth at the oppoſite Point F *(Fig. 23.)*; it would alſo in this Caſe have arrived at the Point M from whence it ſet out; for the exceſs of Velocity it would have gained in F, by its Tendency towards the Earth in its way thither, over and above that, with which it was projected from M, would be ſufficient to carry it off again from the Earth, till it arrived at M; and to make it deſcribe the Path FPM exactly ſimilar and equal to the former, loſing in its way from F to M juſt ſo much Velocity, as it gained by paſſing from M to F; and thereby it would be inabled to perform an infinite Number of Revolutions in the ſame Curve, without requiring a ſecond Projection.

From

From hence it follows, that ſuppoſing a Body projected from a Point at any Diſtance within the Sphere of the Earth's Attraction, with a Force ſufficient to carry it half round without falling to the Surface, it is impoſſible it ſhould fall upon any Part of the other half; but will return to the Point from whence it ſet out, making continual ſucceſſive Revolutions in the ſame Curve; provided it meets with no Reſiſtance from the Medium through which it paſſes, nor any other Obſtacle to obſtruct its Motion *.

From hence alſo it is clear, that, the nearer the revolving Body approaches to the Earth, the faſter it moves; its Velocity being continually increaſed during the Time of its Acceſs towards the Earth, and as much retarded during its Receſs from it. And this Acceleration and Retardation will always be ſuch, that the Body will deſcribe equal Areas in equal Times: the Meaning of which is, that if we imagine a Line conſtantly extended from the Center of the Earth to the Center of the Body, that Line will always deſcribe or paſs through equal Surfaces or Spaces in equal

* Gravity is here ſuppoſed to be inverſely as the Squares of the Diſtances from the Earth, for 'tis poſſible that the Force by which a Body tends towards another, may vary in ſuch a Manner at different Diſtances, that the projected Body ſhall deſcribe a Spiral Line, continually approaching to, or receding from that about which it revolves.

Times,

Times, for it constantly becomes shorter the faster it moves, and *vice versâ* *.

And for the same Reason that a Body, projected with a sufficient Velocity, may by the Force of Gravity be made to describe a Curve round the Earth, and perform continual successive Revolutions therein; it follows that the Moon, may by the same Force of Gravi-

* *Dem.* Let the Time in which the Body performs one Revolution be divided into equal Parts, in the first of which let the Body describe the right Line *AB* (*Fig.* 24.) in the second Part of Time, if not prevented, it would go straight on to *c*, describing the Line *Bc* equal to *AB* by the first Law of Nature; the Lines *SA*, *SB*, *Sc* being drawn, the Triangles *SBA*, *ScB* will be equal to each other, their Bases *AB* and *Bc* being equal and their Heights *S* the same (38. *Elem.* 1.) When the Body arrives at *B*, let the Centripetal Force by one single Impulse turn it out of the straight Line *Bc* into the Line *BC*; in which let it move on uniformly without receiving a second Impulse till it comes to *C*. Let *Cc* be drawn parallel to *SB* meeting *BC* in *C*; then at the End of the second Part of Time, the Body will be found in *C*, having described the Diagonal of the Parallelogram *Nc* (Chap. 4. §. 2.). Draw *SC*, and the Triangle *SCB* will be equal to the Triangle *ScB*, (each having the same Base *SB* and being between the same Parallels *Cc* and *SB*) and therefore also equal to the Triangle *SBA*. For the same Reason, if the Centripetal Force acts in the Points *C*, *D*, *E* successively, so as to make the Body describe the straight Lines *CD*, *DE*, *EF*, &c. in so many equal Parts of Time, the Triangles *SCD*, *SDE*, *SEF*, &c. will be all equal to one another and to the Triangle *SAB*. Consequently equal Areas are described in equal Times. Let us then suppose the Bases of those Triangles, *viz.* *AB*, *BC*, *CD*, *DE*, &c. diminished *in infinitum*, and likewise the Times in which they are described; then will the Perimeter *A*, *B*, *C*, *D*, *E*, *F*, &c. become a Curve, and any Number of those Triangles taken together, (or their Areas) will be proportionable to the Times in which they are described. *Q. E. D.*

other

ty be made to revolve about the Earth, or any other Planet by the like Force, about the Sun; if the Velocities with which they move are duly adjusted to the Forces by which they are acted upon.

WHEN a Body revolves about another in this Manner, that Force or Power by which it is prevented from flying off (as it otherwise would do in a Tangent to the Curve which it describes) is call'd the *Centripetal*; the counter-action of this, by which it endeavours to fly off, the *Centrifugal*; these, by the third Law of Nature being equal to each other, are called by one common Name, *Central Forces*; that with which the Body is at first projected or continues its Motion from any Point, is the *Projectile* Force; and the Time in which it performs one Revolution, the *Periodical* Time.

THESE Forces, properly relating to the Motions of the Heavenly Bodies, will be more largely treated of in another Place,

CHAP. IX.

Of the Communication of Motion.

I. BEFORE we proceed to explain the Laws, by which Bodies communicate their Motion from one to another, it is very necessary

neceſſary to make a Diſtinction between Motion and Velocity; which ought to be well obſerv'd, and is as follows.

By the Motion of a Body (ſometimes called its Quantity of Motion, ſometimes its *Momentum*) is not to be underſtood the Velocity only, with which the Body moves; but the Sum of the Motion of all its Parts taken together: conſequently the more Matter any Body contains, the greater will be its Motion, though its Velocity remains the ſame. Thus, ſuppoſing two Bodies, one containing ten Times the Quantity of Matter the other does, moving with equal Velocity; the greater Body is ſaid to have ten Times the Motion, or Momentum, that the other has: for 'tis evident, that a tenth Part of the larger has as much, as the other whole Body. In ſhort, that Quality in moving Bodies, which Philoſophers underſtand by the Term Momentum or Motion, is no other than what is vulgarly call'd their *Force*, which every one knows to depend on their Quantity of Matter, as well as their Velocity. This is that Power, a moving Body has to affect another in all Actions that ariſe from its Motion, and is therefore a fundamental Principle in Mechanics.

II. Now, ſince this Momentum, or Force, depends equally on the Quantity of Matter a Body contains, and on the Velocity with which it moves; the Method to determine

how great it is, is to multiply one by the other. Thus, ſuppoſe two Bodies, the firſt having twice the Quantity of Matter, and thrice the Velocity which the other has; any two Numbers that are to each other as two to one, will expreſs their Quantities of Matter (it being only their relative Velocities and Quantities of Matter which we need conſider) and any two Numbers that are as three to one, their Velocities; now multiplying the Quantity of Matter in the firſt *viz.* two by its Velocity three, the Product is ſix; and multiplying the Quantity of Matter in the ſecond by its Velocity, *viz.* one by one, the Product is one; their relative Forces therefore or Powers will be as ſix to one, or the Moment of one is ſix Times greater than that of the other. Again if their Quantities of Matter had been as three to eight and their Velocities as two to three, then would their Moments have been as ſix to twenty four, that is, as one to four.

THIS being rightly apprehended, what follows, concerning the Laws of the Communication of Motion by Impulſe, and the Mechanical Powers, will be eaſily underſtood.

The Communication of Motion.

I. In Bodies not Elaſtic.

III. THOSE Bodies are ſaid to be not *Elaſtic*, which, when they ſtrike againſt one another,

ther, do not rebound, but accompany one another after Impact, as if they were joined. This proceeds from their retaining the Impression, made upon their Surfaces, after the impressing Force ceases to act. For all rebounding is occasioned by a certain Spring in the Surfaces of Bodies, whereby those Parts, which receive the Impression made by the Stroke, immediately spring back, and throw off the impinging Body; now, this being wanting in Bodies void of Elasticity, there follows no Separation after Impact.

IV. WHEN one Body impinges on another which is at rest, or moving with less Velocity the same Way, the Quantity of the Motion or Momentum in both Bodies taken together remains the same after Impact, as before; for by the third Law of Nature, the Reaction of one being equal to the Action of the other, what one gains, the other must lose.

THUS, suppose two equal Bodies, one impinging with twelve Degrees of Velocity on the other at rest: the Quantities of Matter in the Bodies being equal, their Moments and Velocities are the same; the Sum in both twelve; this remains the same after Impact (§. 4.), and is equally divided between them (§. 3.); they have therefore six apiece, that is, the impinging Body communicates half its Velocity, and keeps half.

V. WHEN two Bodies impinge on each other by moving contrary Ways, the Quantity of Motion they retain after Impact, is equal to the Difference of the Motion they had before; for by the third Law of Nature, that which had the least Motion, will destroy an equal Quantity in the other, after which they will move together with the Remainder, that is the Difference.

THUS for Instance, let there be two equal Bodies moving towards each other, the one with three Degrees of Velocity, the other with five, the Difference of their Moments or Velocities will be two; this remains the same after Impact (§. 5.) and is equally divided between them (§. 3.) they have therefore one apiece: that is, the Body, which had five Degrees of Velocity, loses three or as much as the other had, communicates half the Remainder, and keeps the other half*.

* From these Positions it is easy to deduce a Theorem, that shall shew the Velocity of Bodies after Impact in all Cases whatever. Let there be two Bodies A and B, the Velocity of the first a of the other b; then (§. 2.) the Moment of A will be expressed by Aa, and of B by Bb: therefore the Sum of both will be $Aa + Bb$; and $Aa - Bb$ will be the Difference when they meet. Now these Quantities (by §. 4. and 5) remain the same after Impact; but knowing the Quantities of Motion and Quantities of Matter, we have the Velocity (which §. 3. is the same in both) by dividing the former by the latter (as follows from §. 2.) therefore $\frac{Aa + Bb}{A + B}$ or $\frac{Aa - Bb}{A + B}$ will in all Cases express the Velocity of the Bodies after Impact.

II.

II. In Elaſtic Bodies.

VI. BODIES perfectly *Elaſtic* are ſuch as rebound after Impact with a Force equal to that with which they impinge upon one another: thoſe Parts of their Surfaces, that receive the Impreſſion, immediately ſpringing back, and throwing off the impinging Bodies with a Force equal to that of Impact.

VII. FROM hence it follows, that the Action of Elaſtic Bodies on each other (that of the Spring being equal to that of the Stroke), is twice as much as the ſame in Bodies void of Elaſticity. Therefore, when Elaſtic Bodies impinge on each other, the one loſes, and the other gains twice as much Motion as if they had not been Elaſtic; we have therefore an eaſy Way of determining the Change of Motion in Elaſtic Bodies, knowing firſt what it would have been in the ſame Circumſtances, had the Bodies been void of Elaſticity.

THUS, if there be two equal and Elaſtic Bodies, the one in Motion with twelve Degrees of Velocity impinging on the other at reſt, the impinging Body will communicate twice as much Velocity as if it had not been Elaſtic, that is, (by §. 4.) twelve Degrees, or all it had; conſequently it will be at reſt, and the other will move on with the whole Velocity of the former.

VIII. IT ſometimes happens, that in Bodies not Elaſtic, the one loſes more than half its Velo-

Velocity, in which Caſe, ſuppoſing them Elaſtic, it loſes more than all; that is, the exceſs of what it loſes above what it has, is negative, or in a contrary Direction; thus, ſuppoſe the Circumſtances of Impact ſuch, that a Body which has but twelve Degrees of Velocity, loſes ſixteen; the overplus four is to be taken the contrary Way, that is, the Body will rebound with four Degrees of Velocity. *v. g.* Let it be required to determine the Velocity of a Body after Impact againſt an immoveable Object. Let us firſt ſuppoſe the Object and Body both void of Elaſticity: 'tis evident the impinging Body would be ſtopt or loſe all its Motion, and communicate none; if they are Elaſtic, it muſt loſe twice as much (by §. 7.) and conſequently will rebound with a Force equal to that of the Stroke.

IX. It is ſufficient if only one of the Bodies is Elaſtic, provided the other be infinitely hard; for then the Impreſſion in the Elaſtic Body will be double of what it would have been, had they both been equally Elaſtic: and conſequently the Force with which they rebound, will be the ſame as if the Impreſſion had been equally divided between the two Bodies.

X. There are no Bodies, that we know of, either perfectly Elaſtic, or infinitely hard; the nearer therefore any Bodies approach to Perfection of Elaſticity, ſo much the nearer do

do the Laws, which they obſerve in the mutual Communication of their Motion, approach to thoſe we have laid down.

XI. Sir ISAAC NEWTON made trials with ſeveral Bodies, and found that the ſame Degree of Elaſticity always appeared in the ſame Bodies, with whatever Force they were ſtruck; ſo that the Elaſtic Power in all the Bodies he made trial upon, exerted it ſelf in one conſtant Proportion to the compreſſing Force. He found the Celerity with which Balls of Wool bound up very compact, receded from each other, to bear nearly the Proportion of five to nine to the Celerity wherewith they met; and in Steel, he found nearly the ſame Proportion; in Cork the Elaſticity was ſomething leſs; but in Glaſs much greater; for the Celerity, with which Balls of that Material ſeparated after Percuſſion, he found to bear the Proportion of fifteen to ſixteen to the Celerity wherewith they met *.

XII. WE have hitherto ſuppoſed the Direction, in which Bodies impinge upon one another, to be perpendicular to their Surfaces: when it is not ſo, the Force of Impact will be leſs, by how much the more that Direction varies from the Perpendicular; for it is manifeſt that a direct Impulſe is the greateſt of all

* Newt. Princip. Phil. pag. 21.

others

others that can be given with the ſame Degree of Velocity *.

XIII. This is the Caſe, when Bodies impel one another by acting upon their Surfaces; but in Forces, where the Surfaces of Bodies are not concerned, as in Attraction *&c.* we muſt not conſider the Relation which the Direction of the Force has to the Surface of the Body to be moved, but to the Direction in which it is to be moved by that Force. Here the Force of Action will be leſs, by how much the more theſe two Directions vary from each other †. My Meaning in both

* The Force of oblique Percuſſion is to that of direct, as the Sine of the Angle of Incidence to the Radius.

Dem. Let there be a Plane as *AD* (*Fig.* 25.) againſt which let a Body impinge in the Point *D* in the Direction *BD*: which Line may be ſuppoſed to expreſs the Force of direct Impulſe, and may be reſolved into two others (Chap. 4. §. 2.) *BC* and *BA*; the one parallel, the other perpendicular to the Plane; but that Force which is exerted in a Direction parallel to the Plane can no Way affect it; the Stroke therefore ariſes wholly from the other Force expreſſed by the Line *BA*; but this is to the Line *BD*, as the Sine of the Angle of Incidence *ADB* to the Radius; from whence the Propoſition is clear.

If the Surface of the Body to be ſtruck is a Curve, then let *AD* be made a Tangent to *D* the Point of Incidence, and the Demonſtration will be the ſame.

† The Force of oblique Action is to that of direct, as the Co-Sine of the Angle comprehended between the Direction of the Force, and that wherein a Body is to be moved thereby, to the Radius.

Dem. Let *FD* (*Fig.* 26.) repreſent a Force acting upon a Body as *D*, and impelling it towards *E*; but let *DM* be the only Way in which it is poſſible for the Body to move; the

Force

Fig. 22. P. 44.

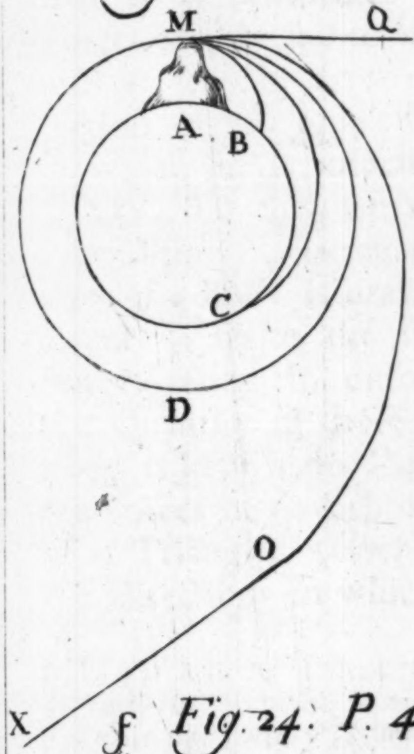


Fig. 23. P. 44.

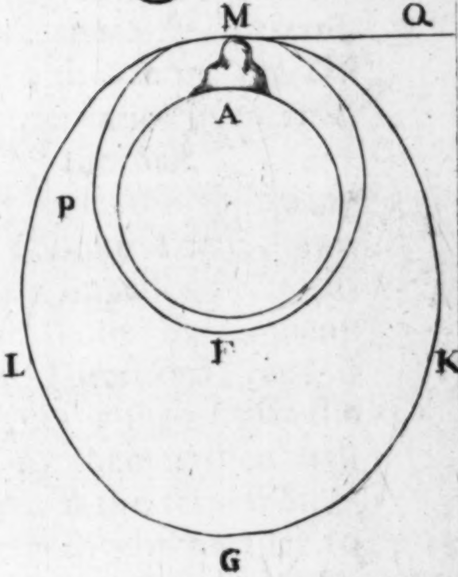


Fig. 24. P. 47.

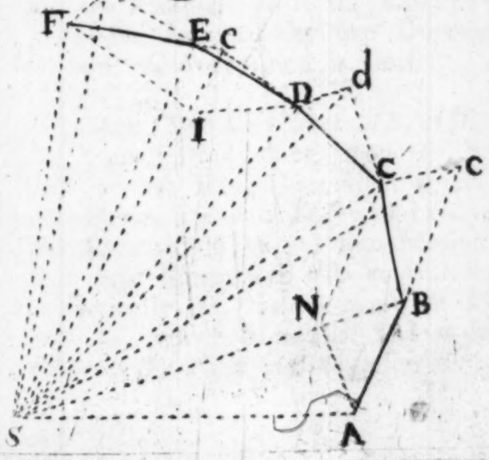
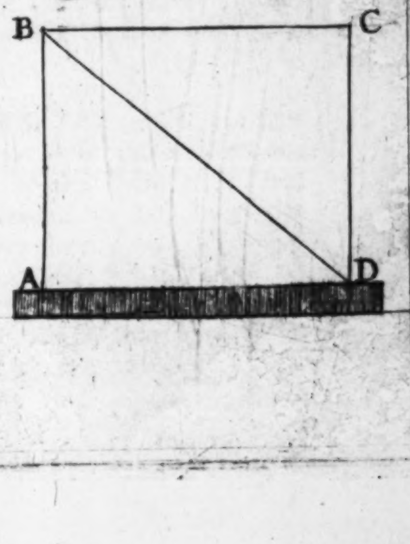


Fig. 25. P. 56.



Cases will be underſtood from the Inſtance of a Ship under Sail. The Force by which the Wind acts upon the Sail, will be leſs, by how much the more its direction varies from one that is perpendicular to its Surface: but the Force of the Sail to move the Ship forward, will be leſs, by how much the more the Direction of the Ship's Courſe varies from that, in which ſhe is impell'd by the Sail.

XIV. To this we may add the following Propoſition, relating to oblique Forces, *viz.* that, if a Body is drawn or impelled three different Ways at the ſame Time by as many Forces acting in different Directions; and if the Quantity of thoſe Forces is ſuch, that the Body is kept in its Place by them: then will the Forces be to each other, as the ſeveral Sides of a Triangle, drawn reſpectively parallel to the Directions in which they act *.

Force *FD* may be reſolved (Chap. 4. §. 2.) into two others *FG* and *FH*, or which is equal to it *GD*; but 'tis evident that only the Force *GD* impels it towards *M*. Now, *FD* being the Radius, *GD* is the Co-Sine of the Angle *FDG* comprehended between the two Directions *FE* and *GM*; from whence the Propoſition is clear.

* *Dem.* Let the Lines *AB*, *AD*, *AE*, (*Fig.* 27.) repreſent the three Forces acting upon the Body *A* in thoſe Directions and by that Means keeping it at reſt in the Point *A*. Then the Forces *EA* and *DA* will be equivalent to *BA*, otherwiſe the Body would be put into Motion by them (*contra Hypoth.*) But theſe Forces are alſo equivalent to *AC* (Chap. 4. §. 2.) conſequently *AC* may be made uſe of to expreſs the Force *AB*; and *EC*, which is parallel and equal to *AD*, may expreſs the Force *AD*, while *AE* expreſſes its own: but *ACE* is a Tri-

CHAP. X.

Of the Mechanical Powers.

I. HAVING, in the foregoing Chapter, accounted for the Communication of Motion by Impulſe; we proceed next to conſider Motion as communicated by Means of certain Inſtruments, commonly known by the Names of *Mechanical Powers.* The Uſe of theſe Powers conſiſts chiefly in managing great Weights, or performing other Works with a determinate Force.

II. THEY are uſually reckoned five. *viz.* The Lever, the Wheel and Axis, the Pully, the Screw, and the Wedge; to which ſome add the Inclined Plane. To theſe all Machines how complicated ſoever are reducible.

III. THESE Inſtruments have been of very ancient Uſe; for we find that *Archimedes*, was well acquainted with the extent of their Power; as may be inferred from that celebrated Saying of his, Δὸς ποῦ στῶ, καὶ τὴν γῆν κινήσω. By which he meant, that the greateſt imaginable Weight might be moved with the ſmalleſt Power.

angle whoſe Sides are all parallel to the given Directions; therefore the Sides of this Triangle will expreſs the Relation of the Forces by which the Body is kept at reſt. *Q. E. D.*

IV.

IV. That Body, which communicates Motion to another, is called the *Power*; that which receives it, the *Weight*.

V. That Point in a Body, which remains at reſt, while the Body is turning round, is called the *Center of Motion*. Beſides this, there are two other Centers in Bodies, 1. that of *Magnitude*, which is a Point, as near as poſſible, equally diſtant from all the external Parts of the Body; 2. that of *Gravity*, or that, about which all the Parts of the Body, in whatever Situation it is placed, exactly balance each other.

VI. When a Body communicates Motion to another, it loſes juſt ſo much of its own, as it communicates to that other; the Action of one, being equal to the Reaction of the other. See Chapter the laſt §. 4. and 5.

VII. When two Bodies have ſuch Relation to each other (ſuppoſe them fixed to different Parts of the ſame Machine) that if one be put into Motion, the other will thereby neceſſarily have ſuch a Degree of Velocity given it, that their Moments will be equal; it will then be impoſſible, that one ſhould begin to move without communicating to the other a Quantity of Motion equal to its own; 'tis evident therefore from the laſt Propoſition, that if we ſuppoſe it to begin to move, in that very Inſtant it muſt loſe all its own Motion by communicating the whole to the other

Body, and therefore being left to it ſelf, will remain at reſt, and communicate none at all.

Now the Moments of two Bodies are equal (Chap. 9. §. 2.) when the Velocity of the firſt is to that of the ſecond, as the Quantity of Matter of the ſecond to that of the firſt; for if we ſuppoſe their Quantities of Matter as one to three, then by the Suppoſition their Velocities are as three to one; and if we multiply the Quantity of Matter in the firſt *viz.* one, by its Velocity three, and that of the other *viz.* three by its Velocity one; their Products are equal; their Moments are therefore by the Definition (Chap. 9. §. 1. and 2.) equal. They will alſo be equal, when the Spaces the Bodies paſs over are in that Proportion; for the Times they both move in being the ſame, the Spaces will always be as the Velocities.

VIII. From hence it follows, that in any Machine, whether ſimple or compound, the Power however ſmall may have a Moment equal to that of the Weight; provided the Machine be ſuch, that when it is in Motion, the Velocity of the Power ſhall bear ſuch Proportion to that of the Weight, as the Weight does to the Power; for then, what the Power wants in Quantity of Matter or Weight, will be made up in Velocity; conſequently their Moments will be equal by §. the laſt, and therefore by §. 7. they will exactly balance each other; or be in *Æquilibrio.*

IX. But if the Power bears a greater Proportion to the Weight, than the Velocity of the Weight to that of the Power; it will then have a greater Momentum than the other, and consequently may communicate such a Momentum to it as it will receive, without losing all its own; the Remainder therefore, if sufficient to overcome the Friction of the Machine, will put it into Motion.

We proceed now to treat of each Mechanical Power in its Order, and

I. Of the Lever.

X. The Lever is a right Line (or Bar whose Weight in Theory is not considered) moveable on a Center, which is call'd its *Fulcrum*, or *fixed Point*.

XI. The Æquilibrium in this Machine is, when the Distance of the Power from the fixed Point is to that of the Weight from the same, as the Quantity of Matter in the Weight is to that in the Power.

For, supposing the Lever placed on its Fulcrum with the Weight to be raised at one End, and the Power applied to the other; 'tis evident, the farther the Power is placed from the Fulcrum or Center of Motion, the larger will be its Sweep when the Machine is put in Motion; that is, it will move over so much more Space in the same Time than the Weight to be raised: now, if it is placed just

ſo much farther from the Fulcrum, as it is leſs than the Weight, it will move juſt ſo much faſter; their Moments therefore will be equal (§ 7.) and conſequently the Power and Weight will exactly balance each other, or be in Æquilibrio *. And, if the Power is ſufficiently augmented to overcome the Friction of the Machine, it will put it in Motion.

THE Lever is of three Kinds. 1. When the fixed Point is between the Weight and the Power, as in the laſt Caſe. 2. When the Weight is between the fixed Point and the Power. 3. When the Power is between the fixed Point and the Weight.

IN all which Caſes the Æquilibrium will be, when their Diſtances from the fixed Point are ſuch, that their Velocities ſhall be inverſely as their Quantities of Matter; for then by (§. 7.) being at reſt, neither of them will communicate any Motion to the other.

THE common Scales may be conſidered as

* Geometrically thus. Let *AB* (*Fig.* 28.) repreſent the Lever, *F* the Fulcrum, *W* the Weight, *P* the Power, the one ſuſpended at the Extremity of the Lever *A*, the other at *B*, and let *BF* be to *FA* as *W* to *P*; then while the Lever moves from the Situation *AB* into that of *CD*, the Point *B* which ſuſtains the Power will move as much farther than *A* which ſuſtains the Weight (and conſequently as much faſter ſince they perform their Motions in the ſame Time) as the Arch *BD* is longer than *AC*; that is, the Triangles *BFD* and *AFC* being ſimilar, as the Arm *BF* is longer than *AF*, which (*ex Hypoth.*) is as much as the Weight exceeds the Power, they will therefore (§. 7.) be in Æquilibrio. *Q. E. D.*

a Lever

a Lever of the firſt Kind, where the Weight and Power are applied at equal Diſtances from the fixed Point.

THE Steelyard is alſo a Lever of the firſt Kind, but whoſe Arms are unequal.

THE Difference between the Uſe of the Scales and the Steelyard conſiſts in this; that as in one you make uſe of a larger Power (or more Weights) to eſtimate the Weight of an heavier Body; in the other you uſe the ſame Power, but give it a greater Velocity with reſpect to that of the Weight by applying it farther from the fixed Point, which by §. 7. will have the ſame Effect.

II. The WHEEL and AXIS

XII. THIS Machine is a Wheel, that turns round together with its Axis; the Power in this is applied to the Circumference of the Wheel, and the Weight drawn up by means of a Rope wound about the Axis.

XIII. IN this there will be an Æquilibrium, when the Weight is to the Power, as the Diameter of the Wheel to the Diameter of the Axis.

'TIS evident, the Velocity of the Power will exceed the Velocity of the Weight, as much as the Circumference of the Wheel exceeds that of its Axis; becauſe the Spaces they paſs over in one Revolution will be as thoſe Circumferences; that is, as much as the Diameter

meter of one exceeds that of the other, (the Circumferences of Circles being as their Diameters;) what therefore in this Case the Power wants in Weight will be made up in Velocity, from whence (§. 7.) there will be an Æquilibrium *.

THE Use of this Machine is to raise Weights to greater Heights than the Lever can do, because the Wheel is capable of being turned several Times round, which the Lever is not; and also to communicate Motion from one Part of a Machine to another; accordingly there are few compound Machines without it.

III. The PULLEY.

XIV. A Pulley is an Instrument composed of one or more Wheels moveable on their Axes.

XV. A simple Pulley, if its Axis is fixed, is of no other use, than to alter the Direction of the Power; for the Power and Weight will both move through an equal

* Geometrically thus. Let *AB* (*Fig* 29) be the Diameter of the Wheel, *DE* that of the Axis, *W* the Weight, and *P* the Power; when the Wheel begins to move, the Point *B* and *D* will describe similar Arches about the Center *C*, in the same Manner the Point *A* and *B* in the Lever were shewn to do about the fixed Point *F* (*Fig.* 28.) that is the Point *B* will move as much faster than *D*, as *CB* is longer than *CD* or *AB* than *DE*, the Motion therefore of *P* (§ 7) will be equal to that of *W*. From whence the Proposition is clear.

Space

Space in the ſame Time. But in a Pulley not fixed. as in *Fig.* 30. where the Rope runs under it, or in a Combination of Pullies as in *Fig.* 31. the Æquilibrium will be, when the Power is to the Weight, as one to the Number of Ropes, that paſs between the upper and lower Pullies.

For ſuppoſe one End of the Rope fixed in B *(Fig.* 30.*)* the other ſupported by the Power P, it is evident, that in order to raiſe the Weight W one Foot, the Power muſt riſe two, for both Ropes *viz.* BC and CP, will be ſhortened a Foot apiece, whence the Space run over by the Power, will be double to that of the Weight; if therefore the Power is to the Weight as one to two, their Moments will be equal: for the ſame Reaſon if there be four Ropes paſſing from the upper to the lower Pullies as in *Fig.* 31. the Velocity of the Power will be quadruple to that of the Weight, or as four to one, *&c.* In all Caſes therefore when the Power is to the Weight, as one to the Number of Ropes paſſing from the upper to the lower Pullies, (§. 7.) there will be an Æquilibrium.

XVI, If the Pullies be diſpoſed as in Figure the 32*d*, each having its own particular Rope, the Action of the Power will be very much increaſed; for here every Pully doubles it, wherefore the Power is four Times greater with two Pullies, eight Times with three,

ſixteen Times with four *&c.* For, it is evident from the Conſideration of the Figure, the firſt will move half as faſt as the Power, the ſecond half as faſt as that, and ſo on; wherefore (§. 7.) the Power is doubled by each Pulley.

THE uſe of the Pulley is nearly the ſame with that of the Wheel and Axis, but it is more portable, and eaſier to be fixed up.

IV. The SCREW.

XVII. IN this Machine the Æquilibrium will be, when the Power is to the Weight, as the Diſtance between any two contiguous Threads or Spirals in the Screw, to the Way deſcribed by the Power in one whole Revolution. It is manifeſt from the Form of the Machine *(Fig. 33.)* that in one Revolution of the Screw, the Weight will be moved through a Space equal to the Diſtance of two contiguous Threads, and that the Power will run through a Space equal to the Compaſs it takes in one Revolution, therefore (§. 7.) if the Weight exceeds the Power in this Proportion, there will be an Æquilibrium.

THIS Machine is of great Force, and very uſeful in retaining Bodies in a compreſſed State, becauſe it will not run back, as the three foregoing will, when the Power is removed. This ariſes from the great Friction of

of thoſe Parts in the Screw, which during its Motion ſlide upon thoſe, that are at Reſt.

V. The Wedge.

XVIII. This Inſtrument is formed by two equal Rectangles, joined at their lower Baſes, and ſeparated at their upper ones, by a third; which is called the *Back* of the *Wedge*; the other two, its *Sides*.

XIX. In the foregoing Mechanical Powers we have all along conſidered the Weight, as moved in the ſame Direction with that, in which it is acted upon by the Machine, as is commonly the Caſe; but in this, the Weight is generally applied in ſuch a Manner, as to be made to move in a Direction different from that, in which it is protruded by the Wedge; hence it is, that Mathematicians have widely differed in their Determination of the Power of this Machine, ſome conſidering the Weight as moved by it in one Direction, and ſome in another. Nay, there are ſome, even among the late Writers, that have been led into manifeſt Errors by it. We will therefore lay down the ſeveral Proportions, they have given us, for the determining the Power of this Machine, and examine them one by one. 1. It is demonſtrated by ſome, that the Power will be equivalent to the Reſiſtance of the Weight, when it bears ſuch Proportion to it, as the Breadth of the Back of the Wedge,

does to the Sum of its Sides; or, which is the ſame Thing, as half that Breadth to one of its Sides. 2. Others make it ſomewhat larger, and demonſtrate, that it ought to be, as half the Breadth of the Back to the perpendicular Height of the Wedge. 3. Some are of Opinion, that there will not be an Æquilibrium in this Machine, unleſs the Power is to the Weight, as the whole Breadth of the Back to the perpendicular Height. WALLIS, KEIL &c. 4. GRAVESANDE in his *Elements* (L. I. Ch. 13.) gives us the ſame Proportion with the laſt; and in his *Scholium de ligno findendo*, tells us, that when the Parts of the Wood are ſeparated no farther than the Wedge is driven in, the Æquilibrium will be, when the Power is to the Reſiſtance, as half the Breadth of the Back of the Wedge to one of its Sides.

THOSE, who lay down the firſt Proportion for determining the Power of this Machine, ſuppoſe the Parts, which are ſeparated from each other thereby, to recede from their firſt Situation in Directions perpendicular to the Sides of the Wedge. Thus let A C B *(Fig. 34)* repreſent a Wedge; P, P, two Bodies to be ſeparated by it, the one to be moved towards I, the other towards F, in the Directions C I and C F perpendicular to A C and C B; then 'tis evident, that when the Wedge is driven in to the Situation M N O, the two Bodies will be moved to Q and Q; that is, one will

have

Fig. 26. *P.* 56.

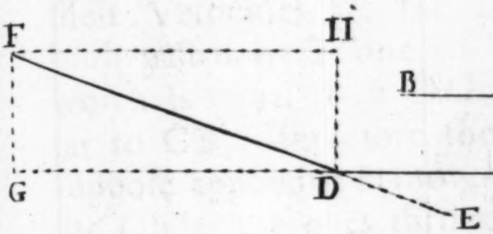


Fig. 27. *P.* 57.

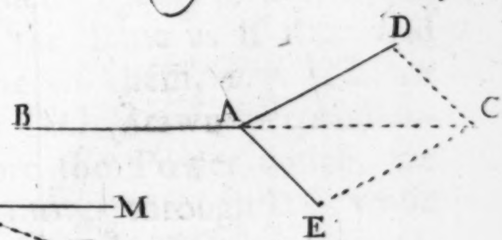


Fig. 28. *P.* 62.

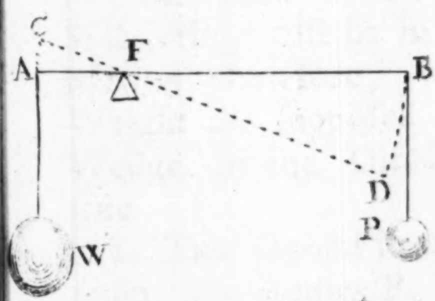


Fig. 29. *P.* 64.

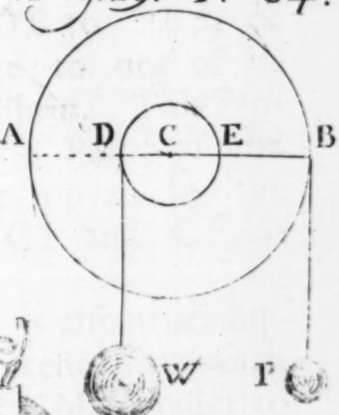


Fig. 30.
P. 65

Fig. 31.
P. 65

have paſſed through the Space CK, the other through CL, but theſe Spaces being equal, their Velocities are the ſame as if they had both paſſed over one of them, *v.g.* CL, or which is equal to it DG (drawn perpendicular to CB); therefore the Power which, we ſuppoſe applied at D moves through DC, while the Obſtacle moves through DG, conſequently (§. 7.) when the Power is to the Weight as DG to DC, that is, as DB to CB*, or half the Back of the Wedge to one of its Sides, they will be in Æquilibrio. This Proportion therefore, when the Parts of the Weight are ſuppoſed to be moved by the Wedge in the Directions CI and CF, is true.

2. THE ſecond Proportion is alſo true, ſuppoſing the Bodies P, P, to recede from each other in the Directions CN, CM, parallel to AB the Back of the Wedge; for when the Wedge is driven in between them, to the Situation MNO, the Bodies will have moved through a Space as CN, or which is equal to it DB, half the Back of the Wedge, and the Power through a Space equal to its Height as before; conſequently (§. 7.) in this Caſe, the Æquilibrium will be, when the Power is

* For (8. *Elem.* 6.) the Triangles *DCG* and *DCB* are ſimilar, and conſequently *DG* : *DC* : : *DB* : *CB*

to

to the Weight, as half the Back of the Wedge to its Height *.

3. THOSE, who imagine there will not be an Æquilibrium, unleſs the Power be to the

* The ſame may be otherwiſe demonſtrated from Section 14. Chapter 9. thus. Let there be a Body as *L* (*Fig.* 35.) drawn againſt the Wedge *ABC* by the Weight *W*, in the Direction *LF*, parallel to the Back of the Wedge *AB*; but prevented from ſliding down towards *C*, by a Plane (whoſe upper Surface we may ſuppoſe repreſented by *EF*) lying under it. I ſay, the Power will be to the Weight, when they are in Æquilibrio, as *DA* to *DC*.

Dem. The Body *L* is here acted upon in three Directions, *viz.* by the Force of the Weight *W* in the Direction *LF*, by the two Planes *CA* and *EF*, in the Directions *LG* and *LI*, perpendicular to their Surfaces; let *GE* be drawn parallel to *LI*, then will the Triangle *LGE* have all its Sides reſpectively parallel to thoſe Directions; conſequently (Chap. 9. §. 14.) if we ſuppoſe *LE* to expreſs the Force of the Weight *W*, *GL* will repreſent the Preſſure of the Body *L* againſt the Wedge; and if that is reſolved into *GE* and *GH* the one perpendicular to the Direction of the Power, the other parallel and contrary to it; the laſt, viz. *GE*, will expreſs the whole Force wherewith the Weight reſiſts the Motion of the Power; but *GE* is to *EL*, as *DA* to *DC* (for the Triangle *EGL* and *DAC* are ſimilar, the Sides of one being *ex Conſtruct.* reſpectively perpendicular to thoſe in the other; v. g. *LG* to *CA*, *EL* to *DC* and *GE* to *DA*); conſequently the Power is to the Weight, when they balance each other, as half the Breadth of the Back of the Wedge to its Height. *Q. E. D.*

Corol. Suppoſe the Body *L* had been drawn againſt the Wedge in the Direction *GL* perpendicular to its Surface, and were to be moved by it in the contrary Direction towards *G*, as in the firſt Caſe; then if *GL* expreſſes the Force with which it is drawn towards the Wedge, *GE* will be that with which it reſiſts the Power; but *GE* is to *GL* as *DA* to *AC*, the Triangles *EGL* and *DAC* being ſimilar; conſequently in this Caſe, the Power will be to the Weight, as half the Breadth of the Back of the Wedge to one of its Sides; as was before demonſtrated.

Weight,

Weight, as the whole Breadth of the Back of the Wedge to its Height, ſuppoſe, as in the laſt Caſe, that the Bodies to be ſeparated, recede from each other in Directions parallel to the Back of the Wedge; and endeavour to ſupport their Opinion by the following Argument: *viz* that, when the Wedge is driven in to the Situation MNO (*Fig.* 34.) as before, each Part of the Weight having moved through a Space equal to half the Back of the Wedge, the whole Weight has therefore moved through twice ſo much, or a Space equal to the whole Back: as much as to ſay, the Whole has moved farther than its Parts; which is abſurd.

4. THIS is GRAVESANDE's Miſtake in his *Elements*, the ſame he has alſo made in his *Scholium de ligno findendo*, and thereby determined the Power in both Places to be twice as big, as it ought to be. If he had proceeded in the following Manner, his Argument would have been eaſier, as well as the Concluſion juſter. Suppoſe the Wedge ABC driven into the Wood QLQ, (as repreſented *Fig.* 36.) which is ſplit no farther than the Point of the Wedge, (or however no farther than is juſt ſufficient to give it room to move) which Caſe GRAVESANDE ſuppoſes in his *Scholium*, I ſay, that in this Situation of the Wedge, the Power is to the Weight, as one fourth Part of the Back of the Wedge to one of its Sides. For it is evident, that when the upper

Ends

Ends of the Wood, which preſs againſt the Wedge in the Points G, H, are put into Motion by the Wedge, they will move in the Directions HI and GF, perpendicular to the Sides of the Wedge, becauſe they turn as it were upon a Joint at L, which we ſuppoſe contiguous to C: again, ſince only the upper Ends of the Wood are put into Motion, and not the lower ones, which remain at L; 'tis evident, that the Motion of each Piece (ſuppoſing their Thickneſs the ſame from End to End, and their Subſtance uniform) will be but half, what it would have been had the lower ones moved with the ſame Degree of Velocity. Now were all the Parts of the Wood to have the ſame Degree of Velocity, the Power would be to the Weight, as in the firſt Caſe, *viz.* as DB to BC (*Fig.* 34.); therefore in this Caſe, it is as half DB to BC, or as one fourth Part of the Back of the Wedge to one of its Sides. Which was to be proved.

XX. THE Form of the *Inclined Plane* being no other than that of half a Wedge, as is manifeſt from the Repreſentation of it (*Fig.* 37.) it follows that what has been demonſtrated of the one, may be applied to the other, and the Properties of both will be found the ſame. For Inſtance, if the Weight W is to be raiſed up the Plane CB, by the Power P, in a Direction parallel to the Plane; inſtead of that, we may ſuppoſe the Weight prevented from

running

Part I. Plate VIII. Pag 72.

Fig. 32. P. 65.

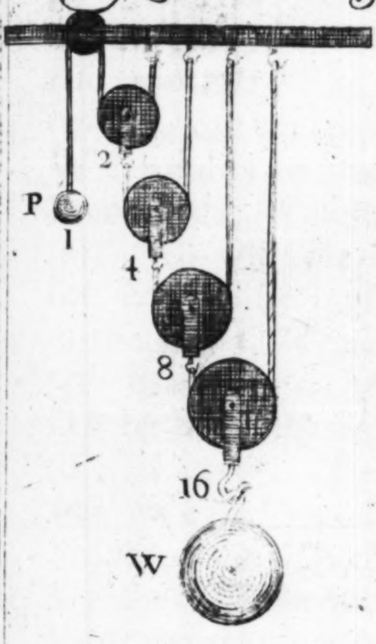


Fig. 33 P. 66

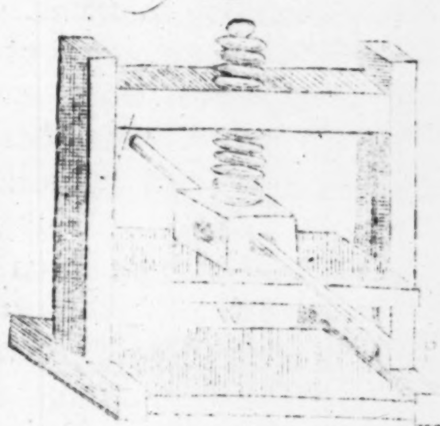


Fig. 34. P. 68

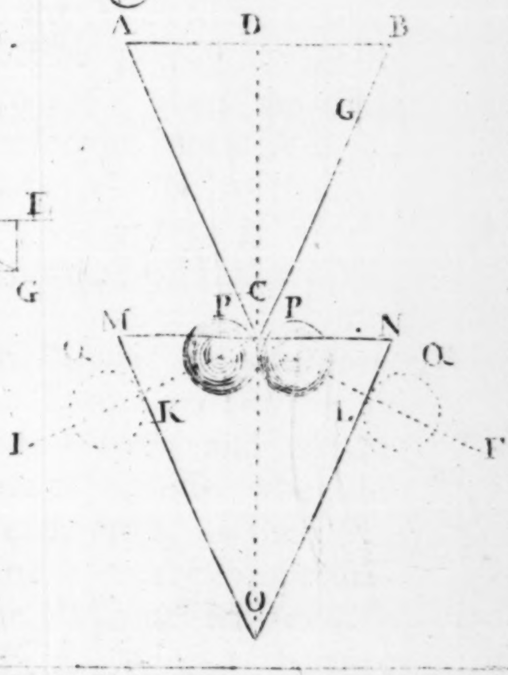
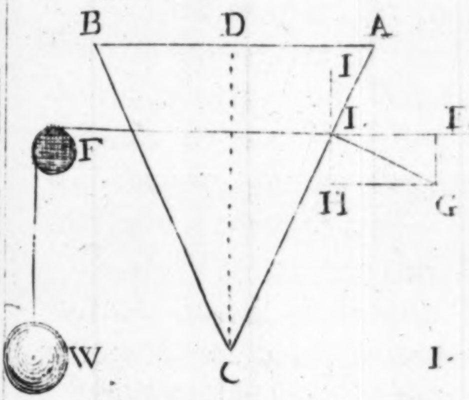


Fig. 35. P. 70.



running off the Plane by the String WB, and the inclined Plane driven under it like a Wedge in the Direction DC: then will the Weight rise towards G in a Direction perpendicular to CB, for we must always suppose the String WB parallel to the Plane, (as it would have been, if the Weight had been drawn up by it;) then will the Action of the Plane upon the Weight be similar to that of the Wedge in the first Case: and consequently the Power will bear such Proportion to the Weight, as DB to BC; that is, as the Height of the Plane to its Length. Again, suppose the Weight was to have been drawn up the Plane by a String in the Direction WF parallel to CD the Base of the inclined Plane; then if the Plane be driven under the Weight as before, it must rise in a Direction perpendicular to CD, that is, parallel to DB: then the Case will be analogous to the second of the Wedge; consequently, the Power will be to the Resistance of the Weight, when there is an Æquilibrium, in the Proportion of DB to DC, as there demonstrated.

XXI. THESE are the Powers or Machines, which under different Forms, constitute all other how complicated soever; and as the Æquilibrium in any one of these is, when the Power and Weight are inversely as their Velocities; so in a Machine however compounded, the Power and the Weight will exactly

balance each other, when they are in this Proportion; for by §. 7. their Moments will then be equal, and the Machine, if at rest, will continue in that State; and if put into Motion by an external Force, will gradually lose it, when that Force ceases to act; on account of the unavoidable Friction of the Machine, and the Resistance of the Air, which it must necessarily meet with, unless its Motion could be performed in a perfect Vacuum. From hence we see the Impossibility of contriving an Engine, whose Motion should be *perpetual*, that is, such as does not owe its continuance to the Application of some external Force; a Problem that has given Birth to an almost infinite Number of Schemes and Contrivances. For unless some Method could be found out of gaining a Force, by the artful Disposition and Combination of the Mechanical Powers, equivalent to that which is continually destroyed by Friction, and the Resistance of the Air, the Motion which was at first given to the Machine, must at length be necessarily lost. But we see, that those Instruments are only different Means, whereby one Body communicates its Motion to another; and not designed to produce a Force which had no Existence before. 'Tis for want of a due Consideration of this, that so many Mechanical Designs have proved abortive, so many Engines unequal to the Performance for which they

Fig. 36. P. 71.

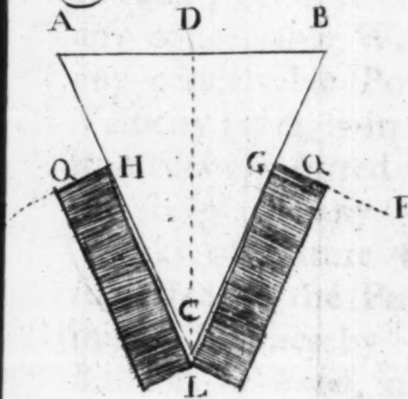
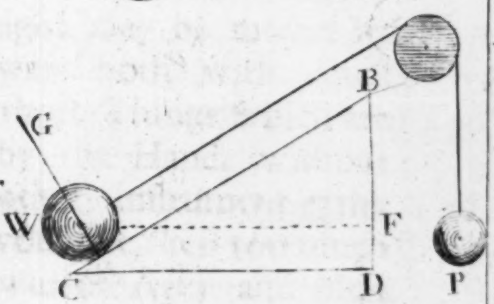


Fig. 37 P. 72.



they were designed, and so many Impossibilities attempted.

" If it were possible, says Bp. Wilkins, " to contrive such an Invention, whereby " any conceivable Weight may be moved by " any conceivable Power, both with equal " Velocity (as it is in those Things which are " immediately stirred by the Hand, without " the help of any other Instrument) the " Works of Nature would be then too much " subjected to the Power of Art; and Men " might be thereby encouraged (with the " Builders of *Babel*, or the Rebel Giants) to " such bold Designs, as would not become a " created Being. And therefore the Wisdom " of Providence has so confined these Human " Arts, that what an Invention hath in the " *Strength* of its Motion, is abated in the *Slowness* of it; and what it has in the extraor" dinary *Quickness* of its Motion, must be al" lowed for in the great *Strength* requisite in " the Power, which is to move it *.

* *Wilkins's* Mathem. Magick. p. 104.